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A TEXTBOOK OF PRACTICAL ASTRONOMY

A TEXTBOOK OF
PRACTICAL ASTRONOMY

Primarily for Engineering Students

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PREFACE

This book is intended for students in civil engineering who have had no other course in astronomy and for other students who wish to do some observational work in practical astronomy. The principal aim has been to present fundamental ideas rather than to include various methods of observing. The simple exercises at the end of each chapter are designed to emphasize these ideas. The data for these exercises are so chosen that the sample pages of the American Ephemeris included in this book are made available for their solution.

If a shorter and more elementary course is desirable, the paragraphs marked with an asterisk may be omitted, also Chapters XII and XIV which are of somewhat more advanced character.

It has been assumed throughout the book that the observer is in the northern hemisphere.

I am indebted to Mr. Sidney McCuskey for his general assistance in the preparation of the manuscript and help with the proof. For the preparation of all the line drawings I am indebted to Professor W. E. Nudd, and to Mr. Frank Herzegh for Figs. 33, 38 and 41. I take pleasure also in expressing my sincere thanks to Dr. John E. Merrill, who has carefully read and criticized the complete manuscript and has offered valuable suggestions.

JASON JOHN NASSAU.

CLEVELAND, OHIO,
December, 1931.

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PRACTICAL ASTRONOMY

CHAPTER I

INTRODUCTION

1. Practical Astronomy.—Practical astronomy deals with the theory and use of astronomical instruments, methods of observing, the reduction of observations, and discussion of astronomical data. That part of practical astronomy, in which we are interested here, deals with the determination of *time, latitude, longitude,* and *azimuth,* and with the instruments used in these determinations.

2. Heavenly Bodies.—Astronomy in general includes the study and description of different bodies, such as the sun and moon, which are commonly known as *heavenly bodies.* These bodies are:

The *stars* that shine in the sky like bright points of light. They are immense bodies giving off both light and heat. To the unaided eye not more than 2,500 of them are visible at one time, but it has been estimated that the 100-in. telescope on Mount Wilson could reveal over one billion. They are so far away that we are hardly able to detect their relative motions, and for this reason they are called *fixed stars.* The nearest star, Proxima Centauri, is twenty-five million million miles away.

Nebulae are composed of great masses of matter, usually incandescent gases, at distances comparable to those of the stars. Some are spherical or elliptical in shape, some spiral, still others quite irregular. Very few are visible to the unaided eye although tens of thousands are known to exist.

The *sun* is a star without which life on the earth would be impossible. It is an average-size star with a mass 333,000 times that of the earth.

The *planets,* nine in number, are opaque spheres revolving about the sun in elliptic orbits with the sun at one of the foci. As viewed from a far-off northern point in space, they move about

the sun in a *counterclockwise direction.* They shine by reflected light from the sun and to the naked eye look much like stars. The earth is one of the planets.

Planets	Mean distance from sun in millions of miles	Mean diameter in miles	Time of revolution around the sun	Number of known satellites
Mercury	35.9	3,009	88 days	None
Venus	67.1	7,575	225 days	None
Earth	92.9	7,918	365 days	One
Mars	141.5	4,216	687 days	Two
Jupiter	483.2	86,700	11.9 years	Nine
Saturn	885.9	76,500	29 5 years	Nine
Uranus	1,782.2	30,900	84 0 years	Four
Neptune	2,792 7	33,000	164 8 years	One
Pluto	3,680 (?)	5,000 (?)	248 (?) years	None

Satellites resemble the planets and revolve about them in elliptic orbits. The Moon, which is 2,160 miles in diameter, is the satellite of the earth and revolves about the earth in a counterclockwise direction in about a month, at an average distance of 239,000 miles from it.

Comets are bodies of small mass and very low density, revolving about the sun in elliptic or parabolic orbits. Like the planets they shine by light from the sun. Bright comets appear in the sky as hazy spots with tails of pale light streaming from them.

Meteors are usually very small bodies weighing but a fraction of an ounce. When they strike our atmosphere they become luminous and remain visible as bright streaks of light for 1 or 2 seconds. The sun, the planets with their satellites, the comets, and the meteors form the *solar system.*

3. The Earth as an Astronomical Body.

1. The earth is nearly spherical and is 7,918 miles in diameter.

2. It rotates on its axis in 24 sidereal hours in a counterclockwise direction (as viewed from a far-off northern point in space). This is the direction from *west* to *east.*

3. Its mean density is about 5.5 times that of water and its mass $6 \cdot 10^{21}$ metric tons.

4. It revolves about the sun in an ellipse at an average speed of 18.5 miles per second. This revolution, like its rotation, is counterclockwise. Its average distance from the sun is 92,900,000 miles. About January 3 it is nearest the sun and its speed is greatest, about July 3 it is farthest from the sun and its speed is least (Fig. 1).

The earth's axis of rotation makes an angle of about $23\frac{1}{2}°$ with the perpendicular to the plane of its orbit. It is clear that when the earth is at *A* (Fig. 1), the sun shines vertically downward on points $23\frac{1}{2}°$ north of the earth's equator. This occurs about June 21 and marks the beginning of summer for

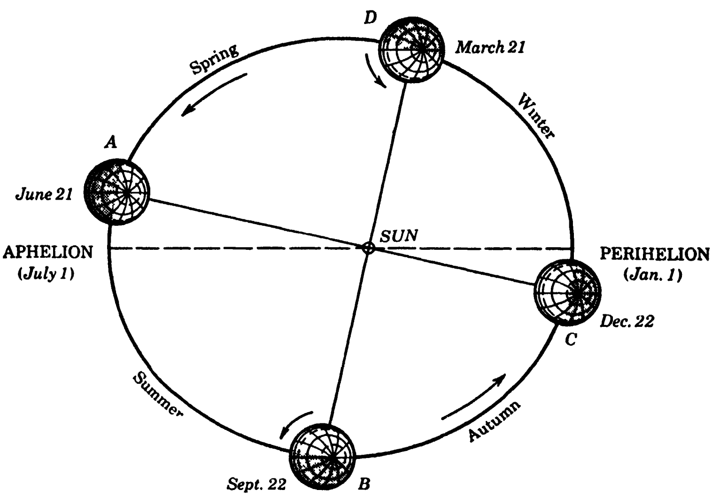

Fig. 1.—The seasons.

the northern hemisphere. About Sept. 22, the earth is at *B*, the sun shines vertically downward on points on the equator, and oblique rays just reach the north and south poles. This position marks the beginning of autumn. Three months later, the earth is at *C* where conditions are opposite to those at *A*, that is, the sun shines vertically downward at points about $23\frac{1}{2}°$ south of the earth's equator. This occurs about Dec. 22 and marks the beginning of winter for the northern hemisphere. On March 21, the beginning of spring, the north and south poles again just receive light as they did at *B*.

4. The Celestial Sphere.—As we look at the heavens on a clear night the stars appear to be fixed on the inner surface of a vast

sphere known as the celestial sphere and we appear to occupy the center of this sphere. In reality, the stars are scattered in space and we, in their midst, project their images on this imaginary sphere. The stars are so remote from the observer that the celestial sphere is assumed infinite in radius, with its center either at the observer on the surface of the earth, or at the center of the earth, or at the center of the sun.

After watching the sky for some time, we see that some stars have disappeared below the western horizon and others have appeared above the eastern horizon, but the relative positions of the stars visible remain the same. Hence we conclude that the celestial sphere apparently rotates on an axis. This rotation of the celestial sphere, making stars rise in the east and set in the west, is due to the rotation, from west to east, of the earth on its axis. The *celestial poles are the two points where the axis of rotation of the earth, extended, pierces the celestial sphere.* Each star appears to describe a circle having its center on the line joining the celestial poles; these circles are known as *diurnal circles.* Figure 3 shows a photograph of arcs of diurnal circles. Work in practical astronomy is immensely simplified by making use of the apparent rotation of the celestial sphere in preference to the actual rotation of the earth.

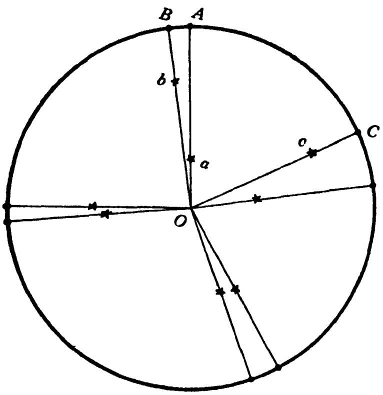

FIG. 2.—Apparent positions of the heavenly bodies. To the observer at O, the heavenly bodies a, b, and c appear on the celestial sphere at A, B, and C, respectively; a and b appear very close to each other, though in reality they are separated by a vast distance.

5. Apparent Path of the Sun among the Stars.—The circle KLM in Fig. 4 represents the intersection of the celestial sphere with the plane of the earth's orbit ABC. Let K, L, and M be the projections of stars on the celestial sphere. S' is the projection of the sun S on the celestial sphere when the earth is at A. Twenty-four sidereal hours later, the earth will be at a position such as B, the stars will appear in the same position as before, but the projection of the sun will be S''. Hence, on account

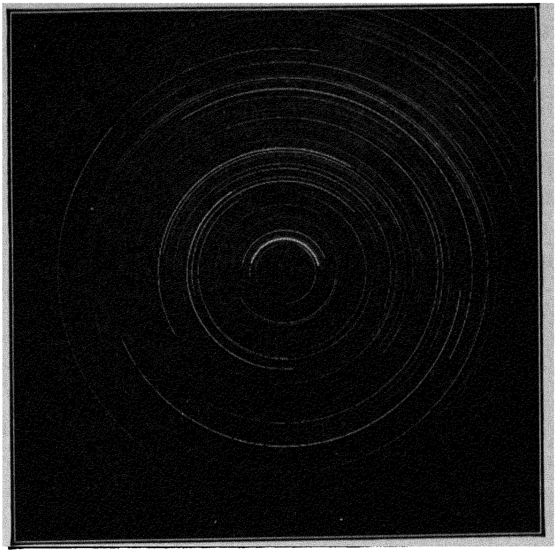

Fig. 3.—Star trails. This photograph illustrates the apparent rotation of the celestial sphere. It was made by pointing a camera in the direction of the north celestial pole, and required an exposure of nearly twelve hours. The arcs of concentric circles are trails of stars. The bright arc near the center was made by the North Star. (*Photographed by Wilson at the Goodsell Observatory.*)

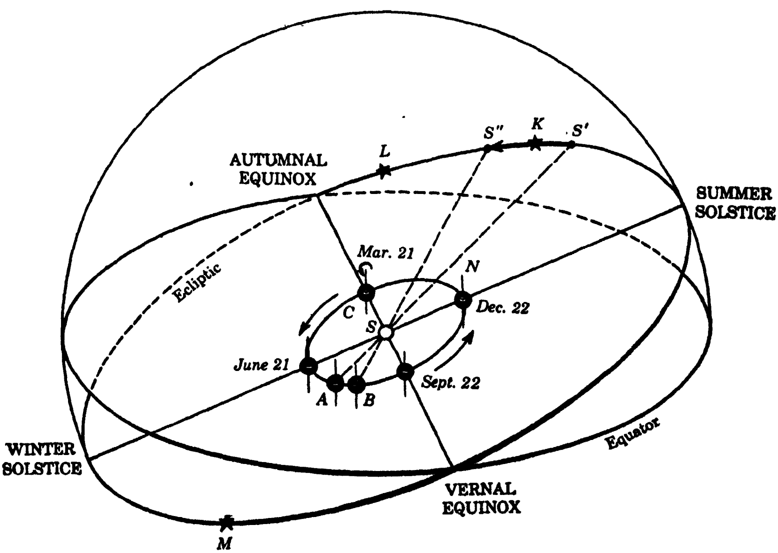

Fig. 4.—The sun projected on the celestial sphere. As the earth moves from A to B, the sun appears to move from S' to S''. This motion is toward the *east* and about 1° *per day*.

of the motion of the earth in its orbit about the sun, the sun appears to move among the stars *from west to east.* Since the earth completes one revolution in about 365¼ days, the apparent motion of the sun among the stars is about one degree per day. The *ecliptic is the intersection of the plane of the earth's orbit with the celestial sphere,* or the great circle described by the sun in its apparent motion during the year.

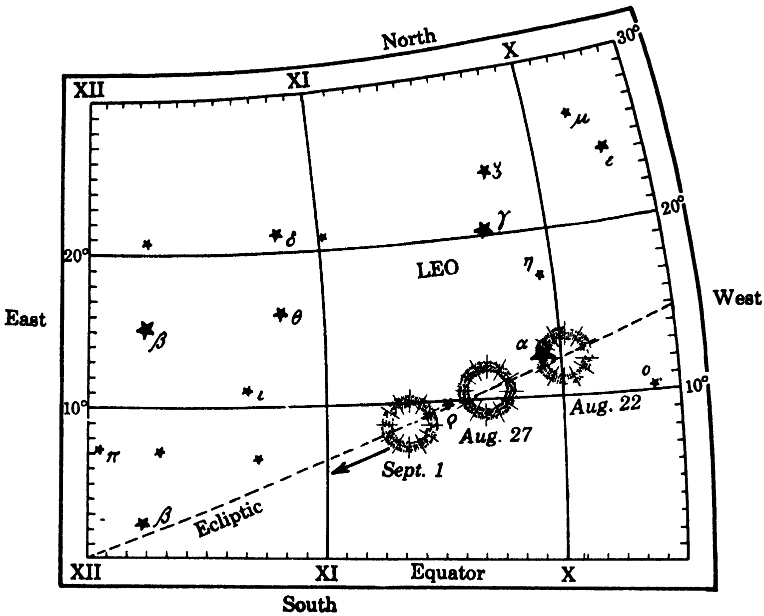

FIG. 5.—Apparent motion of the sun among the stars. The sun has moved in the interval of ten days about 10° toward the *east.*

6. Constellations.—The stars as they appear on the celestial sphere have been divided into groups, which are known as constellations. At present, the entire surface of the celestial sphere is divided into 88 areas or constellations.

Most of the names of the constellations come to us from the ancients. Many names are those of animals, others represent characters in Greek mythology. The stars in a constellation are designated by letters of the Greek alphabet. Usually the brightest star in the constellation receives the letter α, the second brightest β, and so on. For example, in the constellation Ursa Minor (the Little Bear), the North Star is the brightest and is named

α Ursæ Minoris (the genitive case being used); the second brightest is β Ursæ Minoris. A few prominent stars have individual names, *e.g.*, α Lyræ is known as Vega, and β Orionis as Rigel. When the naked-eye stars of a constellation are so numerous as to exhaust the letters of the Greek alphabet, the Roman letters are used. It is apparent that this method will fail in the case of telescopic stars. In this case, a star is referred to by its number in some catalogue. For example, B.1353 means the star so numbered in Boss's "Preliminary General Catalogue."

7. Magnitude of Stars.—Inasmuch as distances of the stars from the observer are different and their intrinsic brightness is different, their apparent brightness is different. To classify the stars according to their brightness, the ancient astronomers adopted an arbitrary scale known as "magnitude" of stars. A modified form of this scale used at present may be explained as follows:

Let a bright star A be one hundred times as bright as a star B; then we assume, in forming our arbitrary scale, that A is five magnitudes brighter than B. Let B represent a star just visible to the unaided eye; such a star is said to be of the sixth magnitude, and hence A is of the first magnitude. Altair and Aldebaran are approximately first magnitude stars. The scale may now be completed as follows:

A star of fifth magnitude is $\sqrt[5]{100}$ or 2.512 times as bright as a star of sixth magnitude.

A star of fourth magnitude is $(\sqrt[5]{100})^2$ or 6.31 times as bright as a star of sixth magnitude.

A star of third magnitude is $(\sqrt[5]{100})^3$ or 15.85 times as bright as a star of sixth magnitude.

A star of second magnitude is $(\sqrt[5]{100})^4$ or 39.81 times as bright as a star of sixth magnitude.

A star of first magnitude is 100 times as bright as a star of the sixth magnitude.

The scale may be extended above and below the limits given. That is, a star of zero magnitude is 2.512 times as bright as a star of first magnitude. Vega is about zero magnitude. Fractional magnitudes may likewise be introduced.

8. Units of Angular Measurements.—The apparent separation of one heavenly body from another is usually measured on

the celestial sphere in degrees, minutes, and seconds (° ′ ″). For example, we say that the distance between Sirius and Procyon is 25°. (The real distance between them is here disregarded.)

Another system of angular measurement which is very convenient in astronomy is that of hours, minutes, and seconds (^{h} ^{m} ^{s}), in which the circle is divided into 24 units called hours.

The relation between the two systems is:

1^{h} corresponds to 15°.
1^{m} (one minute of time) corresponds to 15′.
1^{s} (one second of time) corresponds to 15″.

Tables for conversion from one system to the other are given in many of the logarithmic tables.

Thus 55° 40′ 44″ may be converted to the other system by means of Table I, as follows:

	h	m	s
55° is equivalent to	3	40	0
40′ is equivalent to		2	40
44″ is equivalent to			2.93

55° 40′ 44″ is equivalent to 3^{h} 42^{m} $42^{s}.93$.

CHAPTER II

ASTRONOMICAL SYSTEMS OF COORDINATES

9. Points and Circles of Reference.—To determine the position of a point on the celestial sphere we imagine circles and points of reference on its surface as follows:

1. *The fundamental circle,* an arbitrary great circle of the sphere.
2. *The poles* of this great circle.
3. *Secondary great circles,* the great circles through the poles and therefore perpendicular to the fundamental circle.
4. *The origin,* an arbitrary point on the fundamental circle.

One coordinate of a given point on the celestial sphere is measured from the *origin* along the *fundamental circle* to its intersection with the *secondary circle* through the point; the other is measured on the *secondary circle,* from the intersection to the given point. This system of reference is analogous to the geographic system. The fundamental circle corresponds to the equator, the poles to the north and south poles of the earth, and the secondary great circle to the meridians.

By assuming different fundamental circles we have different systems of coordinates. There are four in common use: the *horizon, equator, ecliptic,* and *galactic* systems. We are mainly concerned here with the first two.

10. The Horizon System.—The point at which a plumb line produced upward pierces the celestial sphere is the *zenith.* The opposite point, below, is the *nadir.* The plumb line does not in general point to the center of the earth (Fig. 10) because the earth is an oblate spheroid and rotates on its shortest axis.

The intersection of the celestial sphere and the plane perpendicular to the line joining the zenith and nadir and half way between these points is the horizon; this is the fundamental circle of this system, the zenith and nadir are the poles.

Since the surface of still water is always perpendicular to the direction of the plumb line, we may define the horizon (for any position of the observer) as the intersection of the celestial

sphere and the plane tangent to the level surface at that point.

Vertical circles are great circles passing through the zenith and nadir; they are secondary great circles of the system.

The vertical circle which passes through the celestial poles (Art. 4) is called the *celestial meridian,* or simply the *meridian.* The *prime vertical* is the vertical circle at right angles to the meridian. The two intersections of the celestial meridian with the horizon are known as the *north* and *south points,* and those of the prime vertical with the horizon, as the *east* and *west points.*

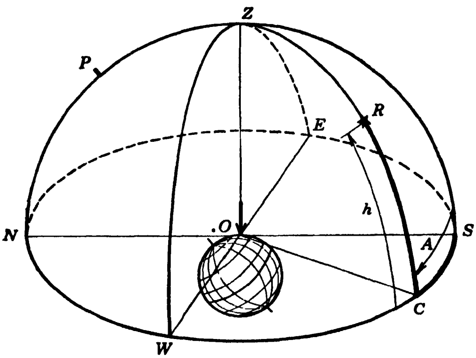

FIG. 6.—The horizon system of coordinates. The observer is at O, with his zenith at Z. P marks the celestial pole, and N and S are the north and south points, E and W the east and west points. ZRC is a vertical circle, through R.

The *azimuth* (A) of a heavenly body is the angular distance measured westward on the horizon from the south point to the
1 2 3
foot of the vertical circle through the body. It is also the angle
4
at the zenith from the meridian westward to the vertical circle through the body.

The *altitude* (h) of a heavenly body is the angular distance measured upward on the vertical circle through the body from the
1 2 3
horizon to the body.
4

Observe that a complete definition of a coordinate of a body must include essentially four things: (a) the circle on which it is measured, (b) the initial point on that circle, (c) the direction of the measurement, and (d) the terminal point.

It is important to note the following:

1. The coordinates of a body in the horizon system are not constant. That is, principally on account of the diurnal motion, the altitude and azimuth of a star continually change.

2. The horizon system is local. That is, the altitude and azimuth of a star at a given instant are different for two observers situated at different places. Just as soon as an observer changes his position, his zenith changes, hence also, his horizon and meridian.

The *zenith distance* (z) of a heavenly body is the arc ZR, Fig. 6, and is equal to the complement of its altitude, *i.e.*, $z = 90° - h$.

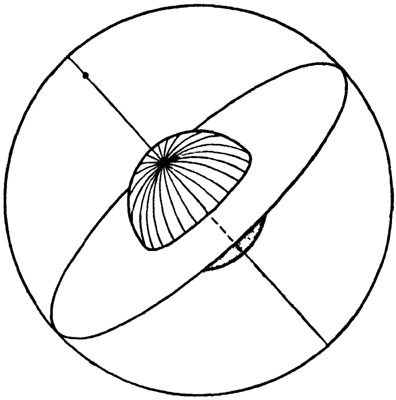

FIG. 7.—The celestial equator is the intersection of the earth's equator with the celestial sphere. The celestial poles are the points of intersection of the earth's axis of rotation produced with the celestial sphere.

Let us consider a perfectly adjusted engineer's transit, properly leveled. Its horizontal plate produced will intersect the celestial sphere in the horizon. When the vertical circle reads 90° the telescope points to the zenith. If the telescope is plunged, the line of sight describes a vertical circle on the celestial sphere. When the zero of the horizontal circle is exactly toward the south point, and the telescope points to a star, the horizontal circle reading will be the azimuth of the star, and the vertical circle reading its altitude.

11. The Equator System.—The *celestial poles* or simply the *poles* have been defined as the points of intersection, with the celestial sphere, of the axis of rotation of the earth produced. The intersection of the celestial sphere and the plane through the center of the earth perpendicular to the line joining the two poles, is the *celestial equator*, *i.e.*, it is the great circle in which the plane of the earth's equator cuts the celestial sphere. Small circles parallel to the equator are known as *parallels of declination* and they are the *diurnal circles*.

The equator is the fundamental circle in the system and the celestial poles are its poles. The secondary great circles are the great circles perpendicular to the equator and are known as

hour circles. The hour circle through the zenith of an observer is the *meridian of the observer.* That is, the meridian of the observer is both a vertical circle and an hour circle.

The points of intersection of the equator and meridian correspond to the north and south points in the horizon system. The one nearest to the zenith corresponds to the south point; we shall name it the Σ-*point.*

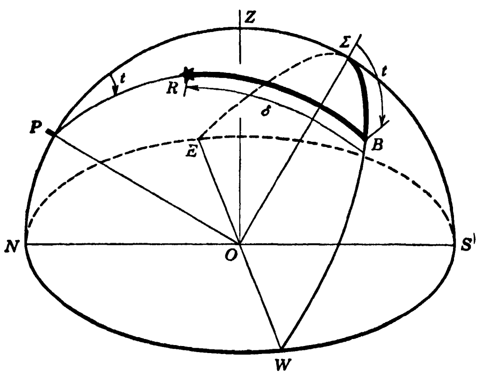

FIG. 8.—The equator system of coordinates. The observer is at O, P is the north celestial pole. Part of the equator is shown as the arc $E\Sigma W$. Arc PRB is part of the hour circle of R.

The *hour angle* (t) of a heavenly body is the angular distance $\underbrace{\text{measured westward}}_{1}$ $\underbrace{\text{on the equator}}_{2}$ $\underbrace{\text{from the point of intersection of the meridian and equator (Σ-point),}}_{3}$ $\underbrace{\text{to the foot of the hour circle through the body,}}_{4}$ or the angle at the pole from the meridian westward to the hour circle through the body. The hour angle is usually expressed in hours. For example, the hour angle of the west point is 90° or 6^h.

The *declination* (δ) of a body is the angular distance measured on the $\underbrace{\text{hour circle through the body}}_{1}$ $\underbrace{\text{from the equator}}_{2}$ $\underbrace{\text{to the body.}}_{3}$ It is $\underbrace{\text{positive when measured northward from the equator and negative when measured southward.}}_{4}$

The hour angle and declination of a heavenly body at a given instant determine its position on the celestial sphere at that instant.

It is important to observe that the coordinates of a star in this system are not constant. The hour angle continually changes since it is measured from a point on the meridian of the observer, the Σ-point, which is not carried along in the diurnal rotation.

12. Equinoxes and Solstices.—To obtain a system in which the coordinates are not affected by the diurnal motion, we assume a point on the celestial sphere which is carried along by the diurnal rotation and which is therefore fixed (as far as possible) with respect to the stars. This point is called the *vernal equinox* (♈), and is defined as that intersection of the ecliptic (Art. 5) with the equator, at which the sun crosses the equator from south to north. This crossing occurs about March 21.

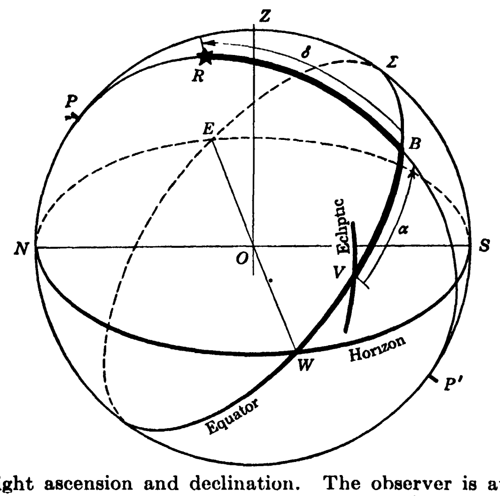

FIG. 9.—Right ascension and declination. The observer is at O, P and P' are the poles of the equator. V represents the vernal equinox, and the arc through it is part of the ecliptic. R is a star.

The other intersection of the ecliptic and the equator is known as the *autumnal equinox*. The sun crosses that point about Sept. 22. The points on the ecliptic midway between the equinoxes are the solstices (Fig. 4). The *obliquity of the ecliptic* (ϵ) is the angle between the planes of the equator and the ecliptic; the magnitude of this angle is about $23\frac{1}{2}°$ and varies slightly, its current value being given in the American Ephemeris.

13. The right ascension (α) of a heavenly body is the angular distance $\underbrace{\text{measured eastward}}_{1}$ $\underbrace{\text{on the equator}}_{2}$ $\underbrace{\text{from the vernal}}_{3}$

equinox to the foot of the hour circle through the body. Right
4
ascension is usually expressed in hours.

The right ascension and declination of a star remain practically constant for years, hence they are well adapted for defining the position of a body on the celestial sphere.

14. The astronomical latitude (ϕ) **of an observer** is the *angle between the direction of the plumb line and the plane of the earth's equator.* Latitude is positive when measured north and negative when measured south of the equator.

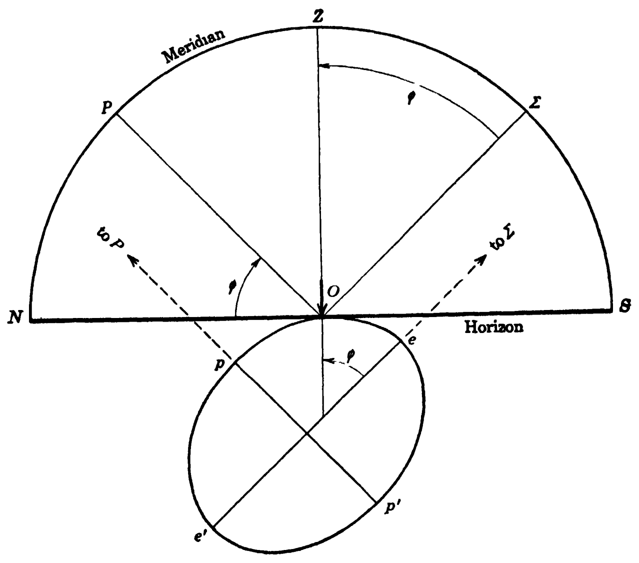

FIG. 10.—Astronomical latitude. The altitude of the celestial pole P measures the latitude of the observer at O.

The ellipse $pep'e'$ in Fig. 10 represents the terrestrial meridian of the observer at O. The direction of the plumb line makes the angle ϕ with the line $e'e$, where e and e' are points on the terrestrial equator. Since ZO is perpendicular to the horizon NS, and OP is perpendicular to ee', angle NOP is equal to ϕ. That is, the *altitude of the pole is equal to the latitude of the observer.* Figure 10 also suggests another definition of latitude as the zenith distance of the Σ-point.

15. Meridian Zenith Distance.—When a heavenly body crosses the meridian of the observer its altitude is the greatest and its zenith distance is the least. The passage of a heavenly body across the meridian of the observer is known as its *transit* over the meridian, or its *culmination.* When the body crosses that part of the meridian which is nearest to the zenith, it is said to be at *upper transit* or *upper culmination;* when it crosses the part farther from the zenith, *lower.*

At a given place there are certain stars that have both upper and lower culmination above the horizon; such stars are known as *circumpolar* for that place. For example, the stars in Fig.

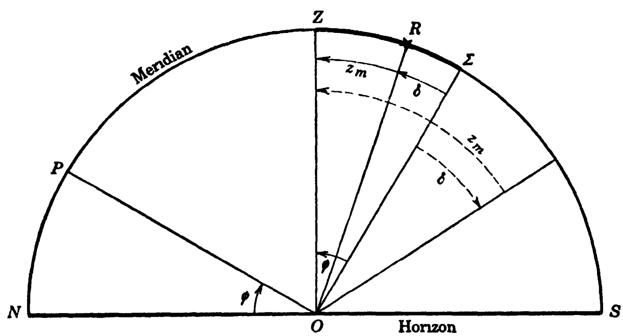

FIG. 11. $\boxed{z_m = \phi - \delta.}$ The relation between the latitude of a place, the *meridian* zenith distance of a body, and its declination, when the heavenly body is south of the zenith.

3 are circumpolar where the photograph was taken. Many observations of heavenly bodies are made at the meridian and a simple relation between the *latitude of the observer*, the *declination of the body*, and *its meridian zenith distance* will be given here.

Suppose in Fig. 11, SZN represents the meridian of the observer and R a star just crossing the meridian, then:

$$\Sigma Z = \text{latitude of observer} = \phi.$$
$$RZ = \text{meridian zenith distance of star} = z_m.$$
$$\Sigma R = \text{declination of star} = \delta.$$

and

$$\boxed{z_m = \phi - \delta \text{ for star south of the zenith}} \qquad (1)$$

In case the declination of a star is negative the same relation holds true, provided δ is substituted in the equation with its proper sign.

When the declination of the star R (Fig. 12) is greater than the latitude of the observer the star will cross the meridian north of the zenith. In this case we have:

$$\boxed{z_m = \delta - \phi \textbf{ for upper transits north of zenith}} \qquad (2)$$

When the star crosses the meridian below the pole we have:

$$\phi = NR' + R'P = 90^\circ - z_m' + 90^\circ - \delta';$$

that is,

$$z_m = 180^\circ - (\phi + \delta) \text{ for stars at lower transit.} \qquad (3)$$

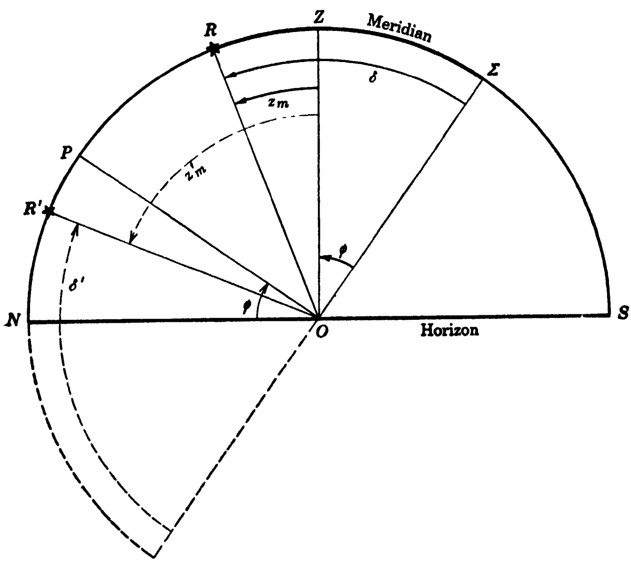

FIG. 12. $\boxed{z_m = \delta - \phi.}$ The relation between the latitude of a place, the *meridian* zenith distance of a body, and its declination, when the heavenly body is north of the zenith and above the pole.

16. Geographic longitude (λ) is the angular distance measured on the terrestrial equator from the intersection of a fixed meridian
1 2
and the equator to the foot of the meridian through the observer.
3
The meridian through Greenwich (England) is usually taken as the fixed meridian. Longitude is reckoned positive westward
4

and negative eastward. It is expressed in hours by astronomers and in degrees by navigators. That is, the longitude of Cleveland is $+81° 34'$ or $+5^h 26^m 16^s$.

17. A summary of the systems of coordinates explained is given in the table below.

	The earth	The celestial sphere		
		Horizon system	Equator system	
			A	*B*
Fundamental circle	Equator	Horizon	Equator	Equator
Poles	North and south terrestrial poles	Zenith and nadir	Celestial poles	Celestial poles
Secondary great circles	Meridians	Vertical circles	Hour circles	Hour circles
Names of coordinates.	Longitude (λ) and latitude (ϕ)	Azimuth (A) Altitude (h) or zenith distance (z)	Hour angle (t) Declination (δ)	Right ascension (α) Declination (δ)
Origin of first coordinate .	Intersection of meridian of Greenwich with equator	The south point	Intersection of meridian with equator (Σ-point)	Vernal equinox (♈)
Positive direction of first coordinate.	Westward	Westward	Westward	Eastward

18. Astronomical Triangle.—In a great many problems of practical astronomy it becomes necessary to transform from one system of coordinates into another. This involves the solution of a spherical triangle.

When any three points A, B, and C, on the surface of a sphere are joined by arcs of *great* circles, the figure so formed is a *spherical triangle.* The arcs AB, BC, and CA are the sides, and the spherical angles at A, B, and C are the *angles* of the spherical triangle. The sides will be denoted by a, b, and c, and the opposite angle by A, B, and C, respectively. Spherical trigonometry deals with relations between the sides and the angles

of the spherical triangle. To avoid ambiguity it is customary to take the sides each less than a semicircle.

Three most important formulas in the solution of the spherical triangle are:

a. Law of Sines.

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}. \tag{4}$$

b. Law of Cosines.

$$\cos a = \cos b \cos c + \sin b \sin c \cos A. \tag{5}$$

c. Relation between Two Angles and Three Sides.

$$\sin a \cos B = \sin c \cos b - \cos c \sin b \cos A \tag{6}$$

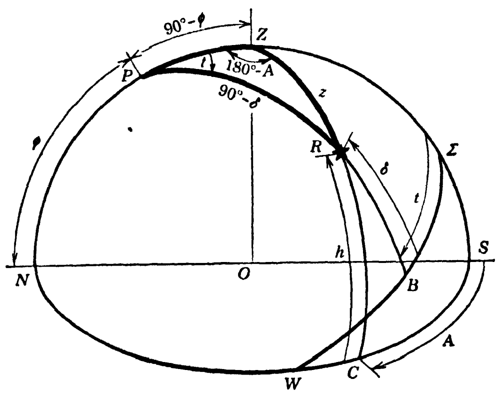

FIG. 13.—Astronomical triangle, with the heavenly body west of the meridian.
1 *Vertices:* North Pole P, the zenith Z, the heavenly body R
2 *Sides:* $PZ = 90° - \phi$
$ZR = z = 90° - h$
$PR = p = 90° - \delta$
3 *Angles:* at P, $= t$, at Z, $= 180° - A$.

From the law of cosines the functions of the half angles in terms of the sides are obtained. The relation for the tangent is,

$$\tan \tfrac{1}{2}A = \pm\sqrt{\frac{\sin (s - b) \sin (s - c)}{\sin s \sin (s - a)}}, \tag{7}$$

where $s = \frac{1}{2}(a + b + c)$.

The spherical triangle having the *pole, zenith,* and a *heavenly body* as the three vertices is called the *astronomical triangle* and is of great importance in practical astronomy. The altitude of the pole P (Fig. 13 or 14) is equal to the latitude of the place, hence, the side ZP of the astronomical triangle is equal to the

co-latitude of the place. The zenith distance ZR is equal to the co-altitude of the star, and the polar distance (p) PR is equal to its co-declination. The angle of the triangle at the pole is equal to the hour angle (t) of the star when the star is west of the meridian (Fig. 13) and $24^h - t$ when the star is east of the meridian (Fig. 14). The angle at Z is equal to $180° - A$ for stars west of meridian and $A - 180°$ for stars east of meridian. The angle at the star R is known as the parallactic angle and

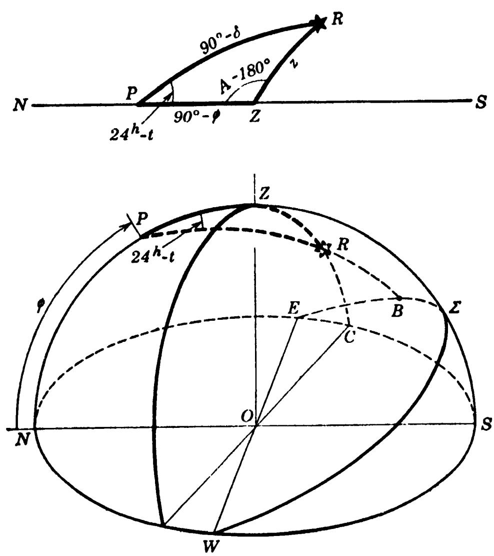

FIG. 14.—Astronomical triangle, with the heavenly body east of the meridian. *Angles:* at P, $= 24^h - t$, at Z, $= A - 180°$.

is not of particular importance here. Observe that, for convenience, the angles of the astronomical triangle have been measured by arcs 90° away from the vertices. That is, *the angle at the pole is measured by an arc of the equator, the angle at the zenith by an arc of the horizon.*

From the law of sines of the triangle PZR, we have, for Fig. 13,

$$\frac{\sin t}{\sin z} = \frac{\sin (180° - A)}{\sin (90° - \delta)}$$

and for Fig. 14

$$\frac{\sin (24^h - t)}{\sin z} = \frac{\sin (A - 180°)}{\sin (90° - \delta)}$$

which gives for both cases,

$$\sin t \cos \delta = \sin z \sin A. \tag{8}$$

From the law of cosines, we may write for Fig. 13

$$\cos z = \cos (90° - \phi) \cos (90° - \delta) + \sin (90° - \phi) \sin (90° - \delta) \cos t$$

and for Fig. 14

$$\cos z = \cos (90° - \phi) \cos (90° - \delta) + \sin (90° - \phi) \sin (90° - \delta) \cos (24^h - t)$$

which gives for both cases,

$$\cos z = \sin \phi \sin \delta + \cos \phi \cos \delta \cos t. \tag{9}$$

Again making use of the law of cosines we have from either figure,

$$\cos (90° - \delta) = \cos (90° - \phi) \cos z - \sin (90° - \phi) \sin z \cos A$$

or

$$\sin \delta = \sin \phi \cos z - \cos \phi \sin z \cos A. \tag{10}$$

Performing the following substitutions in equations (6) and (7):

$\mathbf{A} = t$ or $24^h - t$, $\mathbf{B} = 180° - A$ or $A - 180°$, $\mathbf{a} = z$, $\mathbf{b} = 90° - \delta$, $\mathbf{c} = 90° - \phi$

we have,

$$\sin z \cos A = \sin \phi \cos \delta \cos t - \cos \phi \sin \delta \tag{11}$$

and

$$\tan \tfrac{1}{2} t = \pm\sqrt{\frac{\sin \frac{1}{2}[z - (\phi - \delta)] \sin \frac{1}{2}[z + (\phi - \delta)]}{\cos \frac{1}{2}[z - (\phi + \delta)] \cos \frac{1}{2}[z + (\phi + \delta)]}} \tag{12}$$

Performing the following substitutions in Eq. (6), $\mathbf{A} = 180° - A$ or $A - 180°$ (the A of the right-hand side stands for azimuth), $\mathbf{B} = t$ or $24^h - t$, $\mathbf{a} = 90° - \delta$, $\mathbf{b} = z$, $\mathbf{c} = 90° - \phi$, we have,

$$\cos \delta \cos t = \cos \phi \cos z + \sin \phi \sin z \cos A. \tag{13}$$

The formulas derived above are those most used in this course; others will be derived as they are needed.

19. Orienting the Coordinate Systems on the Celestial Sphere.—It is of great importance that the student should master the systems of coordinates and be able to imagine readily the circles and points of reference on the celestial sphere.

Directly overhead is his zenith, the north pole will be located easily by locating Polaris, the North Star. This is done by having in mind that the altitude of Polaris is approximately equal to the latitude of the observer and that it lies on a line through the pointers of the Big Dipper and 30° from them. (The distance between the pointers is 5°.) There is no other bright star in that region. The great circle through the zenith and Polaris will be his meridian. The south point is now located, and 90° on either side of it along the horizon are the east and west points. The Σ-point is 90° minus the latitude above the south point. A great circle joining the east, Σ, and west points will determine the equator. There is no bright star near the vernal equinox. Imagine it south of the Square of Pegasus, on the equator. The autumnal equinox is approximately half way between the bright stars, Spica and Regulus. Having thus the circles and points of reference, the coordinates (A, h), (t, δ), and (α, δ) of a heavenly body may be estimated.

It is also important to be able to draw figures similar to those given in this chapter. Draw such figures[1] for different latitudes and estimate the coordinates of stars placed on them.

Example: Consider Fig. 14 with $\phi = 60°$. The star at R has $A = 290°$ and $h = 40°$. It is required to estimate t and δ.

We have: $NP = 60°$, hence,

$$S\Sigma = 30°, A = 270° + 20°, \text{ and } CR = 40°.$$

Therefore, by drawing $E\Sigma$ and PB and roughly estimating from the figure we have

$$\Sigma B = 45° \text{ or } 3^h\ 0^m \text{ or } t = 21^h\ 0^m \text{ and } BR = \delta = 25°.$$

A rough solution with the slide rule will give from Eq. (10), $\delta = 25°$; and from Eq. (8), $t = -52°$.

Exercises

1. Give the azimuth and altitude of the west point, the north pole, and the Σ-point.

2. Give the hour angle and declination of the east point, the zenith, and the south point.

3. Give the right ascension and declination of the autumnal equinox and the summer solstice.

4. Give the approximate right ascension and declination of the sun on March 21; also on Dec. 22.

[1] Instead of sketching these figures, they may be drawn on a hemisphere using a spherical protractor. A $4\frac{1}{2}$-in. hemisphere, known as the Willson Hemisphere, and a protractor to fit it are sold by the Eastern Science Supply Company, Boston, Mass.

5. Find α, δ, and t of the east point, at the instant when the vernal equinox is at the west point.

6. Find α and δ and z of the north point, at the instant when the autumnal equinox is at the west point.

7. What is the meridian zenith distance of Sirius ($\delta = -16° 37'$) to an observer whose latitude is 40° N?

8. Give the zenith distance at upper and lower transit of α Ursæ Majoris ($\delta = 62° 8'$) for Washington ($\phi = 38° 55'$).

9. The meridian altitude of Regulus ($\delta = 12° 19'$) above the south horizon is 70°. What is the latitude of the observer?

10. Determine whether or not Capella ($\delta = 45° 56'$) is a circumpolar star in latitude 42° N?

11. What is the approximate meridian altitude of the sun on June 21 at Cleveland (lat. $= +41° 32'$)?

12. The meridian altitude of the sun above the south horizon was observed from a ship at sea, on Dec. 22 and found to be 20°. Find the ship's latitude.

In the following exercises sketch figures similar to Fig. 13 or 14 and estimate from them the required parts. It is, of course, impossible to expect much accuracy from your estimates. Compute the values of these parts from Eqs. (8), (9), and (10). The slide rule or a table of functions to three decimals will be sufficiently accurate for a comparison. If the Willson Hemisphere is used for these exercises, the results will be accurate within 2 or 3 degrees.

13. Find the hour angle and declination of a star, if its azimuth is 50° and its zenith distance 50° at a place in north latitude 40°.

14. Given $\phi = 50°$, $t = 22^h$, $\delta = 0°$. Find A and z.

15. In latitude 30° N, the sun was observed west of the meridian and its altitude was found to be 40°. The declination of the sun at that time was $-10°$. Find its azimuth and hour angle.

16. The latitude of a place is 45° N. Find A and z of Procyon ($\alpha = 7^h 36^m$, $\delta = 5° 24'$), at the instant when the vernal equinox is at the west point.

17. Find the azimuth of Capella ($\delta = 46°$) at the time of setting, at a place of north latitude 35°.

CHAPTER III

TIME—GENERAL PRINCIPLES

20. Measurement of Time.—The unit for measuring time is based on the rotation of the earth on its axis or, as it has been considered in the last chapter, on the diurnal motion of the celestial sphere. This rotation will be assumed to be uniform. To observe this rotation we choose any object in the heavens and note the time interval between two of its successive passages over the meridian of the observer. The object so selected defines the particular kind of time.

21. Sidereal Time.—A *sidereal day is the interval of time between two successive upper transits of the vernal equinox over the same meridian.* On account of the slight westerly movement of the vernal equinox (Precession of the Equinoxes, Art. 31), the length of the sidereal day is slightly less than if it were defined by a fixed point among the stars. This difference is so small that it will be neglected.

The sidereal day is divided into 24 hours and it begins when the vernal equinox crosses the upper meridian of an observer; at that instant the sidereal clock of the observer reads $0^h\ 0^m\ 0^s$ (sidereal noon). When the vernal equinox is at lower culmination the sidereal clock of the observer reads $12^h\ 0^m\ 0^s$ (Fig. 15). *Sidereal time at any instant is the hour angle of the vernal equinox.* That is, if the hour angle of the vernal equinox is 3^h, the sidereal clock of the observer reads 3^h. If the sidereal time is denoted by θ and t stands for hour angle, then by definition,

$$\boxed{\theta = t(\text{of the vernal equinox})} \qquad (14)$$

22. Apparent Solar Time (T_a).—*Apparent solar day* or solar day is the *interval of time between two successive lower transits of the sun's center over the same meridian.* The lower transit is used so that the change of date will occur at midnight.

The apparent solar day is divided into 24 hours beginning with the instant the center of the sun is at lower transit (apparent

midnight). The instant the center of the sun is at upper transit is known as *apparent noon.* Previous to 1925 the usage was to reckon the beginning of the solar day from the instant of its upper transit.

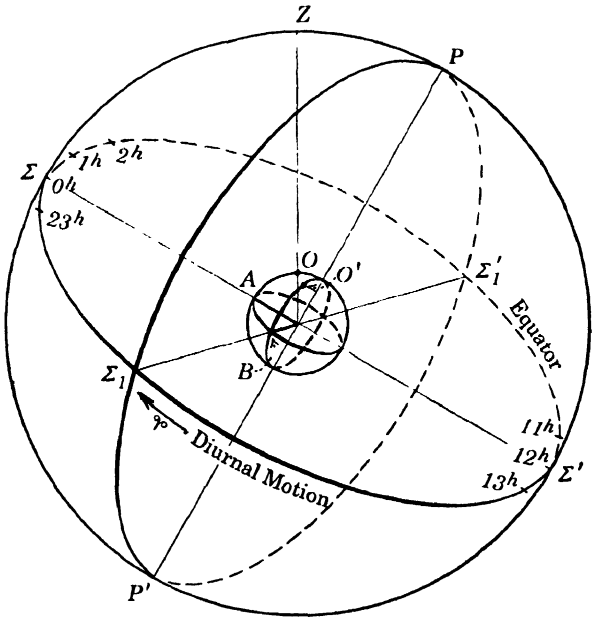

FIG. 15.—Sidereal time. The meridian of the observer at O is the great circle $\Sigma P \Sigma' P'$. The instant the vernal equinox (♈) is at Σ his sidereal clock reads 0^h, and 12^h when ♈ is at Σ', while the sidereal clock of the observer at O', with the meridian $\Sigma_1 P \Sigma'_1 P'$, will read 0^h the instant the vernal equinox is at Σ_1.

Apparent solar time or *apparent time* is the hour angle of the sun's center plus 12^h, or,

$$\boxed{T_a = t(\text{of sun's center}) + 12^h} \qquad (15)$$

For example, when the apparent time is 17^h or 5 P.M., the hour angle of the sun's center is 5^h, and again, the instant when the hour angle of the sun's center is 22^h or -2^h, the apparent time is 10^h or 10 A.M.

It has been stated in Art. 5 that the sun moves eastward among the stars about a degree per day on account of the revolution of the earth about the sun. But the motion of the earth is

not uniform, hence the sun does not move uniformly among the stars. This will cause the apparent solar day to vary in duration. This variation is really due to two causes: (*a*) the slow easterly *non-uniform* movement of the sun, referred to above, which is most rapid when the motion of the earth is most rapid, and (*b*), even if the sun moved uniformly, the length of the solar day could not be constant, for the sun moves *along the ecliptic* and the rotation is measured along the equator. Angles at the celestial pole measuring equal arcs on the ecliptic are not, in general, equal. For example, the angles at the pole P of

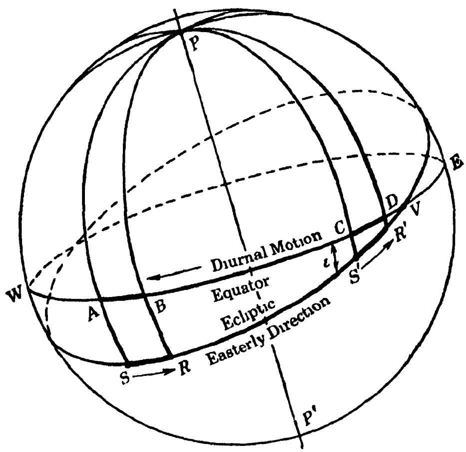

FIG. 16.

the arcs SR and $S'R'$ (Fig. 16) are measured respectively by the arcs on the equator AB and CD. Arc $S'R'$ is near the vernal equinox V and arc SR, equal to $S'R'$, is about 90° from it; it is evident that arc CD is not equal to arc AB.

23. Mean Solar Time or Civil Time.—To avoid the variation of the solar day, a *fictitious sun* is assumed moving *uniformly* toward the east on the *equator* and completing one revolution in the same time that the true sun completes a revolution on the ecliptic. This interval of time (365.2422 mean days) is called the *tropical year*, and is defined as the *interval between two successive passages of the sun through the vernal equinox.* The time given by the fictitious sun or the *mean sun* is such that every day is of exactly the same duration and is equal to the

average solar day. A *mean solar day* or *civil day* is defined as the interval of time between two successive *lower transits* of the mean sun over the same meridian. Previous to 1925 the usage was to reckon the beginning of the mean day from the instant of its upper transit, the astronomical day beginning 12^h earlier. That is, when referring to almanacs previous to 1925, 8 A.M. of Tuesday, June 4, civil reckoning, *is* Monday, June 3, 20^h by the old astronomical reckoning. *Mean solar time* or *civil time* (T) is the hour angle of the mean sun plus 12^h. That is,

$$\boxed{T = t(\text{of mean sun}) + 12^h} \tag{16}$$

Mean noon at any place is the instant of upper transit of the mean sun over the meridian of that place. *Mean midnight* refers to the lower transit. The mean or civil day is divided into 24 hours beginning at midnight. To obtain civil time in which the designations A.M. and P.M. are used, write A.M. after the given mean time when it is less than 12^h; subtract 12^h and write P.M. after the result, when the given mean time is more than 12^h; *e.g.*, 5^h mean or civil time is 5 A.M. and 15^h is 3 P.M.

24. Equation of Time. (E).—Inasmuch as time observations are possible only on the true sun, it will be necessary to introduce a method for changing apparent time to mean or civil time. This is done by a quantity known as the equation of time, which is *defined by,*

$$\boxed{\text{Apparent time} - \text{mean time} = \text{equation of time}} \tag{17}$$

In reality the equation of time is a correction to be applied to either time to obtain the other. It depends on how much the true sun is ahead or behind the fictitious sun. In Fig. 17 the true sun S is behind the fictitious sun M, hence, we must *add* the numerical value of the equation of time to the observed apparent time to obtain the mean time.

Figure 18 shows that the equation of time for the year 1930 is zero on April 16, hence the hour angle of the two suns is the same on this day. The same thing occurs on June 15, Sept. 2, and Dec. 26. The maximum difference between apparent and mean time occurs on Nov. 4. These dates vary a little from year to year. The value of the equation of time is given in many almanacs. In the American Ephemeris (Chap. IV), it is given for 0^h Greenwich Civil Time for every day in the year. The

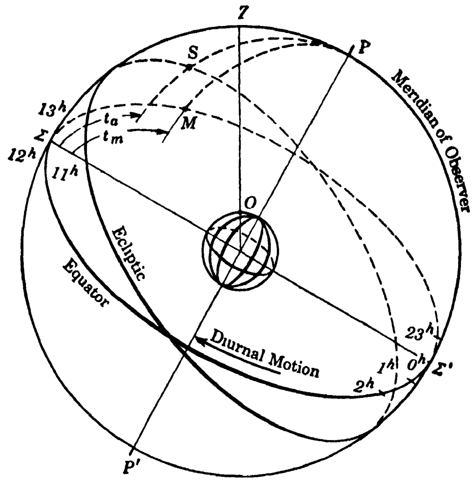

FIG. 17.—Apparent and mean time. The instant the mean sun M is at Σ', the civil clock of the observer at O reads 0^h, and 12^h when M is at Σ. The instant the true sun is at S, the apparent time is $12^h + t_a$ and if the mean sun is at M the corresponding civil time is $12^h + t_m$. While M and S are carried along with the diurnal motion of the celestial sphere, both M and S move about 1° per day in the opposite direction, the first on the equator and the second on the ecliptic.

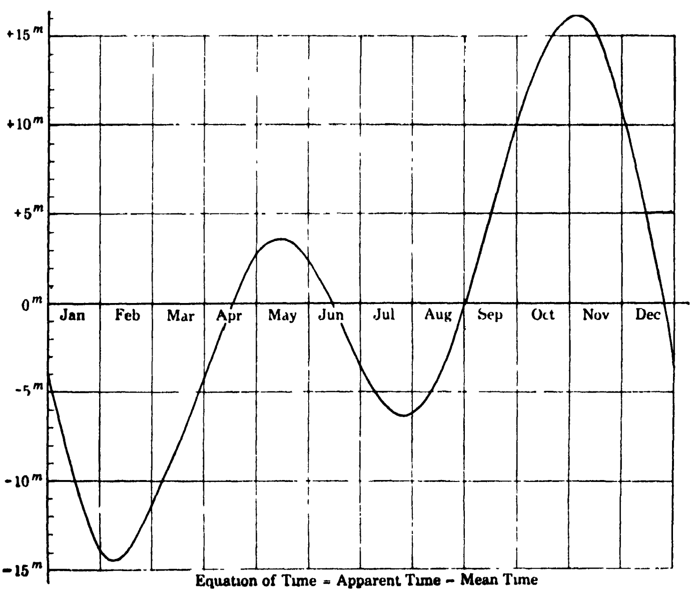

FIG. 18.

problem of converting apparent time to civil time will be taken up in Chap. V, as it is necessary first to know how to use the Ephemeris.

25. Relation between Mean Time and Sidereal Time.—As the mean sun moves eastward on the equator it is evident that at some time during the year it will pass over the autumnal equinox. This occurs each year about Sept. 21. Suppose, for the sake of simplicity, that on Sept. 21, at the instant when the vernal equinox (V, Fig. 19) crosses the meridian of the observer, the mean sun M is exactly on the autumnal equinox,

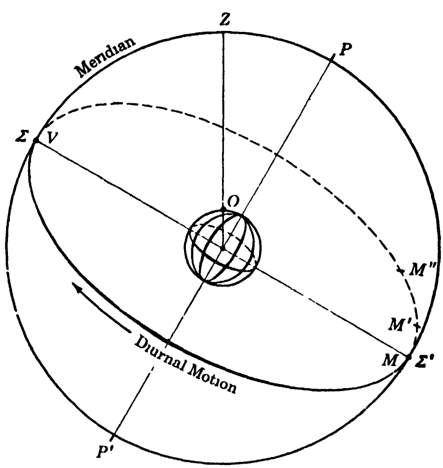

FIG. 19.

hence is just crossing the lower meridian, Σ′. At this instant the sidereal clock of the observer at O will read $0^h\ 0^m\ 0^s$ and the civil clock will read $0^h\ 0^m\ 0^s$.

Twenty-four sidereal hours later, the vernal equinox will again be at Σ, but the mean sun will not have come quite to Σ′, because, during this interval it has moved in a direction opposite to the diurnal motion about 1° or 4^m and will therefore be at M', so that the civil clock will read about $23^h\ 56^m$. From this we observe that the *sidereal clock gains on the civil about* 4^m *per sidereal day.* In a month it will gain 2^h; that is, when the vernal equinox is at Σ on Oct. 21 the sidereal clock of the observer will read zero and the civil clock about 22^h. The mean sun at that instant

will be at M'', where the arc $M''\Sigma' = 30°$ or 2^h. In 6 months, about March 23, the sidereal clock will be 12^h ahead of the civil. At the end of a tropical year (from equinox to same equinox), the mean sun will be again at the autumnal equinox and the sidereal clock will be 24^h or *one day ahead* of the civil clock, *i.e.*, the two clocks will agree again.

The tropical year contains 365.2422 (Art. 23) *mean solar days*, and from what we have just seen 366.2422 *sidereal days.*

$$\text{One sidereal day} = \frac{365.2422}{366.2422} = 0.99726957 \text{ solar day.}$$

$$\text{One mean day} = \frac{366.2422}{365.2422} = 1.00273791 \text{ sidereal days.}$$

The same relations exist between any two time units, for example, one sidereal second = 0.99726957 mean second. Let I represent the number of mean units in a certain *interval* of time, and I' the number of sidereal units in the *same interval*, then,

$$\frac{I}{I'} = \frac{365.2422}{366.2422} = 1 - 0.00273043$$

or

$$\boxed{I = I' - 0.00273043\, I'} \tag{18}$$

and

$$\frac{I'}{I} = \frac{366.2422}{365.2422} = 1 + 0.00273791$$

or

$$\boxed{I' = I + 0.00273791\, I} \tag{19}$$

Equations (18) and (19) convert from one system to the other. For instance, if $I = 24^h$, Eq. (19) will give $I' = 24^h\ 3^m\ 56^s.555$, or a gain in the sidereal clock over the mean of $3^m\ 56^s.555$ sidereal time in 24 mean hours.

Example: If the civil clock reads $0^h\ 0^m\ 0^s$ when the sidereal clock reads $3^h\ 5^m\ 22^s.35$, what would be the reading of the clocks; (*a*) 2 civil hours later, (*b*) 2 sidereal hours later?

a. From Eq. (19) we obtain $I' = 2^h\ 0^m\ 19^s.71$, when $I = 2^h$. Hence, the respective readings of the clocks will be $2^h\ 0^m\ 0^s$ and $5^h\ 5^m\ 42^s.06$.

b. From Eq. (18) we obtain $I = 1^h\ 59^m\ 40^s.34$, when $I' = 2^h$. Hence, the respective readings of the clocks will be $1^h\ 59^m\ 40^s.34$ and $5^h\ 5^m\ 22^s.35$.

In the place of Eqs. (18) and (19), Tables II and III may be used. Also, Tables II and III of the American Ephemeris and Nautical Almanac are for the same purpose and more extensive.

26. Approximate Relation between Sidereal and Civil Time. We have just seen that at sometime during Sept. 21, the sidereal and civil clocks have the same reading. Some years this coincidence occurs in the morning of the twenty-first, other years, in the afternoon, and in still other years, at about 2 A.M. on the twenty-second. This difference depends on the leap year. We will assume that on an average, the coincidence will occur at about 5 P.M.; *i.e.*, on Sept. 21.7. We have seen also that the sidereal clock' gains on the mean $3^m\ 56^s.56$ in 24 mean hours. This gain may be expressed simply, by the quantity $4^m \cdot (1 - \frac{1}{70})$. In D mean days the sidereal clock will gain on the mean $4^m \cdot (1 - \frac{1}{70}) \cdot D$. Hence, if T represents the reading of the mean clock at a given date and θ the corresponding reading of the sidereal clock, we will have,

$$\theta = T + 4^m \cdot (1 - \tfrac{1}{70}) \cdot D, \tag{20}$$

where D now represents the number of days from Sept. 21.7 to the given date.

This formula will yield an accuracy of about two minutes.

Example: When the Greenwich mean clock reads 10^h on May 17, 1929, find the approximate reading of the sidereal clock.

We have, $T = 10^h$; the number of days and tenths of day between Sept. 21.7 and May 17.4 ($10^h = 0^d.4$) is 237.7 days; hence, $D = 237.7$.

Substituting in Eq. (20), we have,

$$\theta = 10^h + 4^m \cdot \frac{69}{70} \cdot 237.7 = 25^h\ 37^m.2,$$

or

$$\theta = 1^h\ 37^m.2.$$

27. Relation between Time and Longitude.—The three kinds of time that we have considered, sidereal, apparent, and mean, have been defined in terms of the hour angle, that is, by means of a quantity measured from the meridian of the observer; hence, they are *local times*. In other words, as soon as an observer changes his position his meridian changes and hence the readings of his clocks must change to conform with the new meridian.

In Fig. 20 let C be west of G and let $\Delta\lambda$ represent the *difference* in their *longitude*, which is an angle measured by the arc BA or $\Sigma_2\Sigma_1$. When the vernal equinox is at V the sidereal clock at G reads t_2, and that of C, t_1. Hence,

$$t_2 - t_1 = \Delta\lambda. \tag{21}$$

Or, the difference in longitude between the two places is equal to the difference in the readings of their sidereal clocks. $\Delta\lambda$ (or its equivalent $t_2 - t_1$) is essentially the *angle at O* and may be expressed in either *degrees* or *hours*.

If we assume the mean sun at V instead of the vernal equinox, the reading of the mean clock at C will be $12^h + t_1$ and that at G, $12^h + t_2$. The difference of the mean times will be $t_2 - t_1$ which is again equivalent to $\Delta\lambda$ and may be expressed in hours or degrees. Therefore it is immaterial what kind of time we are using—sidereal, apparent, or mean—the *difference between the corresponding local times of two places gives their difference in*

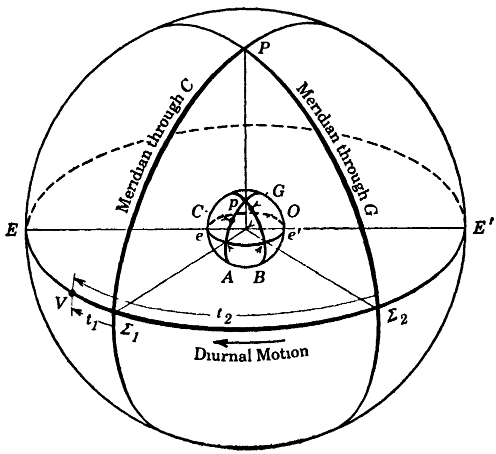

FIG. 20.—Relation between time and longitude. The terrestrial meridians of C and G are respectively pCA and pGB, and their corresponding celestial meridians are $P\Sigma_1$ and $P\Sigma_2$; $eABe'$ is the terrestrial equator, and $E\Sigma_1\Sigma_2E'$ the celestial equator. The instant the vernal equinox V is at Σ_2, the sidereal clock of G reads 0^h and some time later the vernal equinox will be at Σ_1, and the sidereal clock of C will read 0^h. Hence, the clock at G is ahead of the clock at C when C is west of G.

longitude. Observe that in Eq. (21), $\Delta\lambda$ is positive because C is taken west of G, hence t_2, the local time at G, must be greater than t_1, the local time at C; *i.e.*, the *more easterly place will have the later time.* (See Table 2.)

28. Standard Time.—To use local time under the present facilities of travel would be very inconvenient. In order to avoid having different times at practically every city, a system of standard times has been devised. The United States and Canada have been divided into five zones and the times of adjacent

zones made to differ by one hour. Throughout each zone the same time is used and is known as standard time. It is the local civil time of a meridian passing approximately through the center of the zone. Thus the same time is used over a wide area, and this time usually differs from the local time at any place in the zone by less than 30 minutes.

TABLE 1.—TIME ZONES

Standard Meridian west of Greenwich	Zone	Standard Time of zone equal to
60°	Atlantic	Greenwich Civil Time minus 4^h
75°	Eastern	Greenwich Civil Time minus 5^h
90°	Central	Greenwich Civil Time minus 6^h
105°	Mountain	Greenwich Civil Time minus 7^h
120°	Pacific	Greenwich Civil Time minus 8^h

This system of standard time is used practically throughout the world. East of Greenwich, the successive zones have standard time one, two, three, . . . hours *faster* than Greenwich.

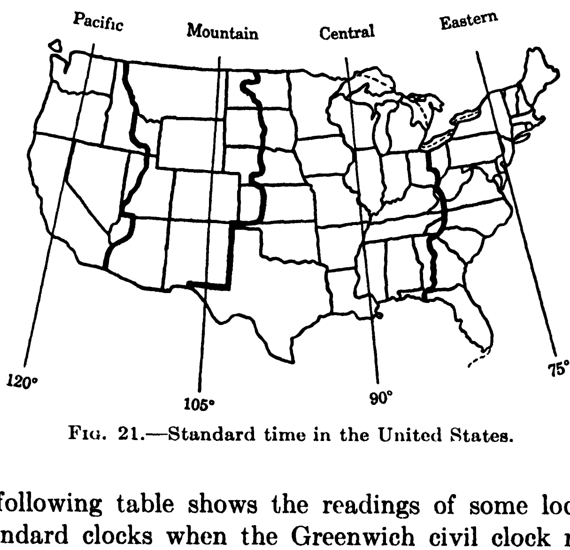

FIG. 21.—Standard time in the United States.

The following table shows the readings of some local, civil, and standard clocks when the Greenwich civil clock reads 12^h (mean noon).

TABLE 2

City	Longitude				Local Civil Time			Standard Time
	h	m	s		h	m	s	
Berlin	0	53	34 80	E	12	53	34 80	1 P.M.
Chicago . .	5	50	26 84	W	6	9	33.16	6 A.M.
Cleveland.	5	26	16.36	W	6	33	43 64	7 A.M.
Denver .	6	59	47 72	W	5	0	12 28	5 A.M.
Paris .	0	9	20.93	E	12	9	20 93	12 noon
San Francisco	8	9	42 86	W	3	50	17 14	4 A.M.
Tokyo	9	18	58.22	E	21	18	58 22	9 P.M.
Washington	5	8	15.78	W	6	51	44 22	7 A.M.

"*Daylight-saving*" time is the usual standard time of a given zone *increased* by one hour. It is used during the summer months in some sections of America and Europe in order that the day's work be done with as little artificial light as possible.

The meridian 180° from Greenwich is known as the *date line;* proceeding westward from Greenwich, the time at that line will be 12 hours slower than that at Greenwich, and proceeding eastward, 12 hours faster. This produces a discontinuity of 24 hours, or one day; therefore steamers sailing west when crossing the date line, omit one calendar day; *e.g.*, if a steamer leaving San Francisco for Japan, crosses the date line on June 25 at 10 P.M., then according to the ship's calendar, 2 hours later June 27 begins. If the crossing is made in the opposite direction a day is repeated.

29. At Any Instant, the Sidereal Time Is Equal to the Right Ascension of Any Heavenly Body Plus Its Hour Angle.—The sidereal time (θ) has been defined as the hour angle of the vernal equinox (V, Fig. 22). The right ascension (α) of a heavenly body S is measured by the arc VB and its hour angle by the arc ΣB. Hence, remembering that α was by definition measured positive toward the east and θ and t positive toward the west, we have,

$$\boxed{\theta = \alpha + t} \tag{22}$$

This equation is true for all positions of S. In a case where $\alpha + t$ is greater than 24^h, θ must be increased by 24^h.

Example: Let the hour angle of S' be required, given its right ascension equal to 7^h and the sidereal time 5^h. Adding 24^h to the given sidereal time, we have 29^h and subtracting 7^h from this we obtain 22^h for the hour

angle. If we substitute directly in Eq. (22), $\theta = 5^h$ and $\alpha' = 7^h$, we obtain $t' = -2^h$, which is of course equivalent to 22^h.

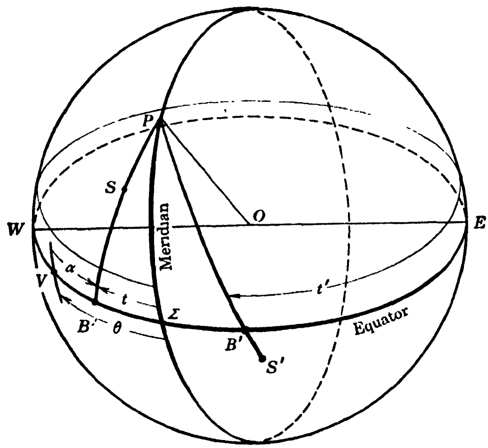

FIG. 22. $\boxed{\theta = \alpha + t}$

When a heavenly body is at upper transit, its hour angle is zero, hence,

$$\boxed{\theta = \alpha, \text{ the instant of upper transit}} \tag{23}$$

and when at lower transit,

$$\theta = \alpha + 12^h. \tag{24}$$

Exercises

1. What is the sidereal time of a place at the instant (*a*) the hour angle of the vernal equinox is 10^h; (*b*) when the autumnal equinox is at upper transit; and (*c*) when the vernal equinox is at the east point?

(The equation of time which is required in the following four exercises may be obtained from Fig. 18.)

2. What is the approximate apparent and civil time at which the sun sets on March 21 (approximate declination of sun, 0°)?

3. Find the civil time of apparent noon for June 1.

4. What is the hour angle of the sun on Nov. 1 when the civil clock of a place reads 7 A.M.? When it reads 1 P.M.?

5. Find the approximate sidereal, apparent, and mean time at sunrise on Sept. 21 (approximate declination of sun, 0°).

6. On what date will the civil clock of a place read 5^h when the sidereal clock reads 10^h?

7. On what date will the civil clock of a place read 22^h when the sidereal clock reads 4^h?

8. What is the approximate reading of the sidereal clock when the mean clo^k reads $13^h\ 12^m$ on Oct. 18?

9. What is the approximate hour angle of the mean sun on Nov. 5, the instant the sidereal clock reads $9^h\ 54^m$?

10. The sidereal clock of a place reads $3^h\ 5^m\ 11^s.2$, when the mean clock reads 0^h.

a. What is the reading of the sidereal clock 3 mean hours later?

b. What is the reading of the mean clock 3 sidereal hours later?

11. The sidereal clock of a place reads $22^h\ 37^m\ 15^s.5$ when the civil clock reads 0^h. What will be the reading of the sidereal clock when, later in the day, the civil clock reads $11^h\ 10^m\ 5^s.0$?

12. The sidereal clock of a place reads $20^h\ 55^m\ 15^s.0$ when the civil clock reads 0^h. What will be the reading of the civil clock when, later in the day, the sidereal clock reads $14^h\ 10^m\ 54^s.0$?

13. The mean clock of a place reads $12^h\ 49^m\ 10^s.0$ when the sidereal clock reads 0^h. What will be the reading of the sidereal clock when, later in the day, the mean clock reads $18^h\ 20^m\ 40^s.0$?

14. What is the reading of the Greenwich mean clock at the instant the mean sun crosses the upper meridian of a place of longitude $8^h\ 50^m\ 10^s.0$ W?

15. What is the sidereal time at Greenwich and at Washington the instant the vernal equinox crosses the upper meridian of Ottawa (long. $5^h\ 2^m\ 51^s.94$ W)?

16. Make a table similar to Table 2 for the following places:

	h	m	s
Princeton (long.	4	58	37.61 W).
Oxford (long.	0	5	0.40 W).
Rome (long.	0	49	56.34 E).
St. Louis (long.	6	0	49.26 W).

17. The right ascension of an equatorial star is 18^h. What will be the reading of the sidereal clock of an observer (*a*) when the star is at upper transit? (*b*) when it is setting?

18. What is the sidereal time of a place at the instant when Arcturus ($\alpha = 14^h\ 12^m$) is at upper transit?

19. The sidereal time of the upper transit of a circumpolar star is $5^h\ 20^m$, its altitude at upper transit is 80°, at lower transit 40°. Find (*a*) the latitude of the place; (*b*) the R.A. and declination of the star.

20. Determine at about what date Fomalhaut ($\alpha = 22^h\ 54^m$) will cross the upper meridian of a place at midnight.

21. On March 21 (app. decl. of sun, 0°), a star of 0° declination rises at sunset. Find the approximate (*a*) R.A. of the star, (*b*) sidereal time of its rising, (*c*) the civil time of its rising, and (*d*) the civil time of its rising on April 21.

CHAPTER IV

SECULAR AND PERIODIC CHANGES—THE AMERICAN EPHEMERIS AND NAUTICAL ALMANAC—STAR CATALOGUES

30. Aberration of Light.—If the earth were stationary, and an observer on it pointed his telescope directly at a star, the rays from the star would come through the tube of the telescope and emerge at the eyepiece. But since the earth is moving, carrying the observer with it, it will be necessary for him to tilt his tele-

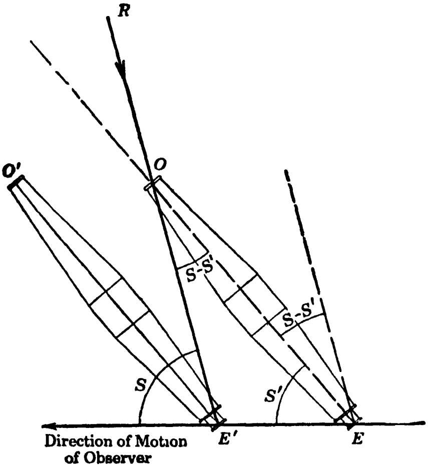

FIG. 23.—Aberration. The telescope does not point exactly in the direction of the star R but is tilted slightly (angle $S - S'$) in the direction of motion of the observer.

scope slightly in the direction of his motion, in order that the rays from the star, after passing the objective, will come to the center of the eyepiece. This apparent displacement of a star is known as *aberration* and is due to the fact that the velocity of light is finite compared to the velocity of the observer. It is here assumed that light consists of material particles and not waves; this is done for the sake of simplicity. It can be shown that waves are affected in the same manner. Let OE

(Fig. 23) be the position of the telescope when the ray of light from R strikes the objective O. The angle that the ray of light makes with the direction of motion of the observer is S, and the angle that the telescope makes with the direction of motion of the observer is S'. Hence the telescope is tilted in the direction of this motion by the angle $S - S'$, in order that at the instant when the observer reaches the position E' the ray of light will reach the same point.

Let t represent the time required for the observer to move from E to E', then,

$$EE' = tv \text{ (where } v \text{ is the velocity of observer)}$$

and

$$OE' = tc \text{ (where } c \text{ is the velocity of light).}$$

If we denote the angle of aberration $S - S'$ by K, we may write from triangle $EE'O$,

$$\frac{\sin K}{\sin S'} = \frac{EE'}{OE'} = \frac{v}{c}.$$

Since K is very small, we write $\sin K = K \sin 1''$, and hence,

$$K = \frac{v}{c \sin 1''} \sin S', \tag{25}$$

which is therefore the correction for aberration, expressed in seconds of arc.

The aberration resulting from the annual revolution of the earth around the sun is called *annual aberration*, and *is included in the star places of the American Ephemeris*. The *diurnal aberration* is caused by the daily rotation of the earth on its axis. This will presently be seen to be a function of the latitude of the observer, and therefore it is different for different places.

**To Compute the Effect of Diurnal Aberration when the Star Appears to Cross the Meridian of the Observer.*—According to Eq. (25) there are two quantities we must consider: the velocity of the observer (v) and the angle (S') that the telescope makes with the direction of his motion. This angle is evidently 90° since the observation is to be made on the meridian.

The observer's velocity in miles per second is,

$$v = \frac{2\pi \cdot R \cos \phi}{24 \cdot 60 \cdot 60},$$

where R is the mean radius of the earth and equal to 3,959 miles, and ϕ the latitude of the observer. Hence, $v = 0.288 \cos \phi$.

Then Eq. (25) gives

$$K = \frac{0.288 \cdot \cos \phi}{186{,}300 \cdot \sin 1''} = 0''.319 \cos \phi = 0^{s}.021 \cos \phi. \qquad (26)$$

Since the value of K is due wholly to the eastward motion of the observer, it is the same at a given latitude for all declinations of stars. It is an eastward displacement perpendicular to the meridian of the observer (Fig. 24).

It is evident that the star R will appear to be on the meridian at a somewhat later time than if the aberration did not exist (Fig. 24). This interval is measured by the hour angle K' of R. From the law of sines in the spherical triangle PAR, and having in mind that the polar distance PR is equal to $90° - \delta$, we have

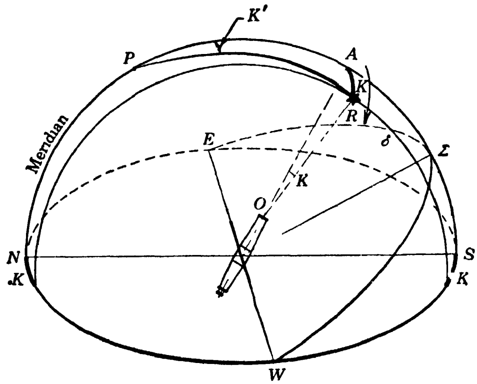

Fig. 24.—Diurnal aberration. On account of the aberration of light the telescope should be tilted in the direction of motion of the observer (from west to east) by the angle K. However, the telescope is permanently mounted on the meridian, hence stars crossing the meridian are observed somewhat later by an amount shown by the angle K'.

$$\frac{\sin K'}{\sin K} = \frac{\sin 90°}{\sin (90° - \delta)}$$

and hence, since K' and K are small angles, we write

$$K' = K \sec \delta.$$

Combining this with Eq. (26) we obtain,

$$K' = 0^{s}.021 \sec \delta \cos \phi \qquad (27)$$

which is the correction expressed in seconds of time, to be subtracted from the observed time of meridian passage of the star.

31. Precession.—The vernal equinox has been defined as one of the intersections of the ecliptic with the celestial equator, and has been used in the definition of the right ascension of stars. But neither of these two circles of reference remains in the same position with respect to the stars; hence both the right ascensions and declinations of the stars change. The motion of the ecliptic in relation to the stars as a whole is so slow that it may be

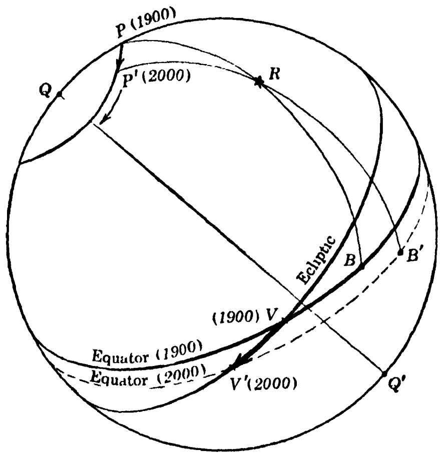

FIG. 25.—Precession of the equinox. Q and Q' are the poles of the ecliptic and P is the north celestial pole in 1900. On account of the precession of the equinox the celestial pole describes approximately a circle about Q; this makes the vernal equinox appear to move westward on the ecliptic effecting a continuous change in the coordinates of the star R.

neglected; the motion of the equator is such that it materially affects the coordinates of the stars and will be considered here.

It is known that the direction of the earth's axis of rotation changes, causing the celestial poles to move among the stars, and therefore, the celestial equator. If Q and Q' (Fig. 25) represent the poles of the ecliptic, the celestial pole P has been found to be describing a circle of about $23\frac{1}{2}°$ radius about the pole Q in 25,800 years. In other words, Polaris will not always be our pole-star, α Draconis was our pole-star some 4,000 years ago, and Vega will be near the north celestial pole some 12,000 years hence. P represents the position of the pole in 1900 and P' in 2000, and V and V' the corresponding positions

of the vernal equinox. This motion of the *vernal equinox on the ecliptic is toward the west and is known* as the *Precession of the Equinox.*

Since the earth's polar diameter is shorter than its equatorial, it may be considered as a sphere with a protuberant ring of matter around the equator. The sun and moon attract this ring and tend to make its plane coincide with the planes of their respective orbits. The rotation of the earth on its axis resists this tendency, with the result that the axis of the earth is made to generate a cone.

As the sun and moon change their positions relative to the earth, the rate of precession varies. For this reason we divide this rate into two parts:

a. Precession, which is a uniform westerly motion of the vernal equinox amounting to about $50''.2$ a year.

b. Nutation, which is the small periodic variation of the motion of the vernal equinox.

32. Definitions.—The right ascensions and declinations of stars are subject to a number of small changes. These changes are divided into two classes.

a. Secular changes are progressive and very slow, requiring long periods of time to complete their cycles; they may be considered proportional to time. *Precession is a secular change.*

b. Periodic changes are those which complete their cycles in a comparatively short time and therefore pass quickly from one extreme value to another. *Nutation is a periodic change.*

After long study of old and new observations it has been found that the "fixed" stars do move on the celestial sphere with reference to their stellar neighbors. This motion of the stars, which tends to change their coordinates slightly, is known as *proper motion.* It is a secular change, and exceedingly small. Relatively few proper motions are known.

When the right ascension and declination of a star are accurately observed by a suitable instrument and the observations reduced to the center of the earth, the place of a star so determined is called *apparent place.* From this definition it is evident that the *apparent place of a star is always used* when such a star is observed for the purpose of determining latitude, time, and azimuth. The *mean place* of a star at a definite epoch is obtained by applying to its apparent place corrections due to precession, nutation, aberration, proper motion and annual parallax.

33. American Ephemeris and Nautical Almanac.[1]—By an ephemeris is meant a catalogue in which the positions (usually, the right ascension and declination) of heavenly bodies are given at equidistant intervals of time. Without such a book of reference astronomical work is almost impossible. The data published are obtained by means of telescopes situated at the principal national observatories. The *American Ephemeris and Nautical Almanac* consists mainly of the following parts:

I. *Ephemeris for the Meridian of Greenwich,* in which the apparent positions of the sun, moon, and planets are given.

II. *Ephemeris for the Meridian of Washington,* in which the mean and apparent positions of 825 stars are given for the instant of their upper transit at Washington, as well as the apparent position of the sun for Washington apparent noon.

III. *Phenomena,* containing predictions of eclipses, occultations, etc., with data for their computation.

Tables, containing data for computing latitude and azimuth from observations of Polaris and tables for converting sidereal time interval to mean solar and *vice versa.*

The Nautical Almanac Office (U. S. Naval Observatory) is also publishing the *American Nautical Almanac*[2] which contains some of the tables of the American Ephemeris. This small book consists *mainly* of the following:

a. Ephemeris for the Meridian of Greenwich, giving the sidereal time of 0^h civil time; the sun's declination, equation of time and semidiameter; the Ephemeris of the moon, Venus, Mars, Jupiter, and Saturn; and the apparent places of 55 stars.

b. Tables containing data for computing latitude and azimuth from observations of Polaris and tables for converting sidereal time interval to mean solar and *vice versa.*

Another useful little table is the *Ephemeris of the Sun and Polaris and Tables of Azimuth and Altitudes of Polaris*[3] which is published by the U. S. Department of the Interior, General Land Office.

[1] The American Ephemeris is published by the United States Government every year, two years in advance of the year of its title, and is sold by the Superintendent of Documents, Government Printing Office, Washington. The price is $1.00. Other books of similar character are *Nautical Almanac* (British), *Berliner Astronomisches Jahrbuch,* and *Connaissance des Temps.*

[2] Sold by the Superintendent of Documents, U. S. Government Printing Office, Washington. Price 15 cents.

[3] Also sold by the Superintendent of Documents. Price 5 cents.

34. To Obtain from Ephemeris Data Relating to the Sun.— Page 44 shows a page from Part I of the American Ephemeris. It gives the apparent right ascension, apparent declination, and semidiameter of the sun, and the equation of time, for 0^h Greenwich Civil Time together with the variations per hour for these quantities. The meaning of the last column will be given in Chap. V. As has been explained in Art. 32, the word *apparent* signifies, "as it appears or as it actually looks from the center of the earth."

To obtain any one of these quantities for a given time, it is evident that an interpolation is necessary between the values for two successive days.

Example: Consider the problem of obtaining the apparent declination of the sun for July 17, 1930, 20^h G.C.T.

We have from the Ephemeris,

Date	App. declination	Var. per hour
h	° ′ ″	″
July 17, 0	21 22 20.6	−24 56
July 18, 0	21 12 20.2	−25 47

I. If we assume that the declination varies proportionally with the time during the 24^h, we obtain for the required declination,

$$21° \ 22' \ 20''\!.6 - (21° \ 22' \ 20''\!.6 - 21° \ 12' \ 20''\!.2) \cdot \frac{20}{24} =$$
$$21° \ 22' \ 20''6 - 8' \ 20''3 = 21° \ 14' \ 0''\!.3.$$

II. If we work with July 17, 0^h using the variation per hour of $-24''56$, we have,

$$21° \ 22' \ 20''\!.6 + (-24''\!.56) \cdot (20) = 21° \ 22' \ 20''\!.6 - 8' \ 11''\!.2 = 21° \ 14' \ 9''\!.4.$$

III. If we work with July 18, 0^h using the variation per hour of $-25''\!.47$ and considering the given time as 4^h *before* July 18, 0^h, we have,

$$21° \ 12' \ 20''\!.2 + (-25''\!.47) \cdot (-4) = 21° \ 12' \ 20''\!.2 + 1' \ 41''\!.9 = 21° \ 14' \ 2''\!.1.$$

This disagreement is due to the fact that the declination does not vary quite in proportion to time during the interval of 24^h and that the given variation per hour is in each case an *instantaneous value* of the variation of the tabulated quantity, that is, a first derivative value for the beginning of each day. Figure 26 clearly shows that in the case of the first interpolation the point lies on the chord; in the second case it lies on the tangent to the curve at the point for 0^h, July 17; and in the third case, on the tangent to the curve at the point for 0^h, July 18. It is therefore evident that, of the three given methods of interpola-

tion, the greatest accuracy is obtained by using the tabulated value for the 0^h *nearest to the given time* with the corresponding variation per hour. For still greater accuracy consult Chauvenet's or Doolittle's "Practical Astronomy" and the illustrations given in the American Ephemeris under the heading, Use of Tables.

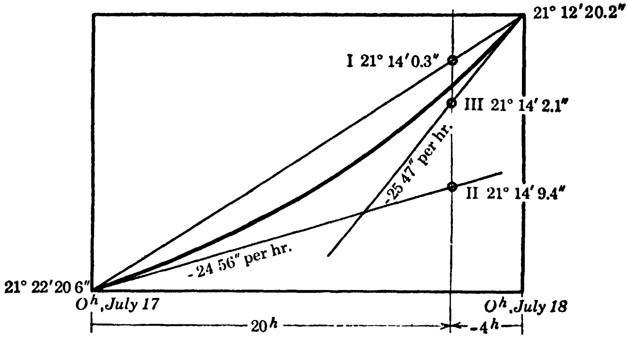

FIG. 26.—Interpolation.

Example: Let the sun's equation of time be required for May 24, 1929, 19^h G.C.T.

We have from the Ephemeris:

Date	Equation of time	Var. per hour
h	m s	s
May 24, 0,	+3 25.04	−0.204
May 25, 0,	+3 19.89	−0.225

Inasmuch as the given time is nearer to 0^h, May 25, we will compute backward from this date.

	m	s
Equation of time at Greenwich 0^h, May 25	+3	19.89
Change in $-5^h = (-0^s.225) \cdot (-5) = \cdots\cdots$	+	1.13
Required equation of time $\cdots\cdots\cdots\cdots\cdots$	+3	21.02

Always have in mind that the required value must lie between the tabulated values given for the two days. In this case, $3^m\ 21^s.02$ is between $3^m\ 25^s.04$ and $3^m\ 19^s.89$.

In the above illustration we have considered the problem for Greenwich time; for any other local time, change the given time of that place into the corresponding time of Greenwich. That is, *to secure from the Ephemeris data relating to the sun for any*

SUN, 1929.

FOR 0h GREENWICH CIVIL TIME.

Date.	Day of the Week.	Apparent Right Ascension.	Var. per Hour.	Apparent Declination.	Var. per Hour.	Semi-diameter.	Hor. Par.	Equation of Time. App.—Mean.	Var. per Hour.	Sidereal Time. (Right Ascension of Mean Sun+12h.)
		h m s	s	° ′ ″	″	′ ″	″	m s	s	h m s
May 17	Fr	3 33 2.77	9.905	+19 10 6.0	+34.39	15 50.63	8.70	+3 46.38	–0.049	15 36 49.16
18	Sa	3 37 0.77	9.928	19 23 41.7	33.57	15 50.45	8.70	3 44.94	0.071	15 40 45.71
19	Su	3 40 59.31	9.950	19 36 57.5	32.74	15 50.26	8.70	3 42.96	0.094	15 44 42.27
20	Mo	3 44 58.39	9.973	19 49 53.2	31.90	15 50.08	8.70	3 40.44	0.116	15 48 38.83
21	Tu	3 48 58.00	9.995	20 2 28.6	31.05	15 49.90	8.69	3 37.39	0.138	15 52 35.39
22	We	3 52 58.15	10.017	+20 14 43.6	+30.19	15 49.72	8.69	+3 33.80	–0.161	15 56 31.94
23	Th	3 56 58.82	10.039	20 26 37.8	29.32	15 49.55	8.69	3 29.68	0.183	16 0 28.50
24	Fr	4 1 0.02	10.061	20 38 11.0	28.44	15 49.38	8.69	3 25.04	0.204	16 4 25.06
25	Sa	4 5 1.73	10.082	20 49 22.9	27.55	15 49.21	8.69	3 19.89	0.225	16 8 21.61
26	Su	4 9 3.95	10.103	21 0 13.4	26.65	15 49.04	8.69	3 14.23	0.247	16 12 18.17
27	Mo	4 13 6.67	10.124	+21 10 42.3	+25.75	15 48.87	8.68	+3 8.06	–0.267	16 16 14.73
28	Tu	4 17 9.89	10.144	21 20 49.4	24.84	15 48.71	8.68	3 1.40	0.288	16 20 11.29
29	We	4 21 13.59	10.164	21 30 34.4	23.91	15 48.55	8.68	2 54.25	0.308	16 24 7.84
30	Th	4 25 17.76	10.183	21 39 57.1	22.98	15 48.39	8.68	2 46.64	0.327	16 28 4.40
31	Fr	4 29 22.39	10.202	21 48 57.4	22.04	15 48.24	8.68	2 38.57	0.346	16 32 0.96
June 1	Sa	4 33 27.47	10.220	+21 57 35.1	+21.09	15 48.09	8.68	+2 30.05	–0.364	16 35 57.52
2	Su	4 37 32.98	10.238	22 5 49.8	20.14	15 47.95	8.68	2 21.09	0.382	16 39 54.08
3	Mo	4 41 38.91	10.255	22 13 41.5	19.17	15 47.81	8.68	2 11.72	0.399	16 43 50.63
4	Tu	4 45 45.24	10.272	22 21 10.0	18.20	15 47.67	8.67	2 1.96	0.415	16 47 47.19
5	We	4 49 51.94	10.287	22 28 15.2	17.22	15 47.54	8.67	1 51.81	0.430	16 51 43.75
6	Th	4 53 59.00	10.301	+22 34 56.8	+16.24	15 47.41	8.67	+1 41.31	–0.445	16 55 40.31
7	Fr	4 58 6.40	10.315	22 41 14.6	15.25	15 47.29	8.67	1 30.47	0.458	16 59 36.87
8	Sa	5 2 14.10	10.327	22 47 8.6	14.25	15 47.18	8.67	1 19.32	0.470	17 3 33.42
9	Su	5 6 22.09	10.338	22 52 38.5	13.24	15 47.07	8.67	1 7.89	0.482	17 7 29.98
10	Mo	5 10 30.33	10.348	22 57 44.3	12.24	15 46.96	8.67	0 56.20	0.492	17 11 26.54
11	Tu	5 14 38.80	10.357	+23 2 25.8	+11.22	15 46.86	8.67	+0 44.28	–0.501	17 15 23.10
12	We	5 18 47.49	10.365	23 6 42.9	10.20	15 46.77	8.67	0 32.16	0.509	17 19 19.66
13	Th	5 22 56.36	10.373	23 10 35.5	9.18	15 46.68	8.66	0 19.86	0.516	17 23 16.22
14	Fr	5 27 5.38	10.379	23 14 3.6	8.16	15 46.59	8.66	+0 7.40	0.522	17 27 12.77
15	Sa	5 31 14.53	10.384	23 17 7.1	7.13	15 46.51	8.66	–0 5.20	0.527	17 31 9.33
16	Su	5 35 23.80	10.388	+23 19 45.9	+ 6.10	15 46.44	8.66	–0 17.91	–0.532	17 35 5.89
17	Mo	5 39 33.16	10.392	23 22 0.0	5.07	15 46.37	8.66	0 30.71	0.535	17 39 2.45
18	Tu	5 43 42.59	10.394	23 23 49.4	4.04	15 46.30	8.66	0 43.59	0.538	17 42 59.01
19	We	5 47 52.08	10.396	23 25 13.9	3.01	15 46.23	8.66	0 56.52	0.539	17 46 55.57
20	Th	5 52 1.59	10.397	23 26 13.7	1.98	15 46.17	8.66	1 9.47	0.540	17 50 52.12
21	Fr	5 56 11.11	10.397	+23 26 48.8	+ 0.94	15 46.11	8.66	–1 22.43	–0.540	17 54 48.68
22	Sa	6 0 20.62	10.396	23 26 59.1	– 0.09	15 46.06	8.66	1 35.38	0.539	17 58 45.24
23	Su	6 4 30.10	10.394	23 26 44.5	1.12	15 46.00	8.66	1 48.30	0.537	18 2 41.80
24	Mo	6 8 39.52	10.391	23 26 5.2	2.15	15 45.95	8.66	2 1.17	0.535	18 6 38.36
25	Tu	6 12 48.88	10.388	23 25 1.2	3.18	15 45.91	8.66	2 13.97	0.531	18 10 34.92
26	We	6 16 58.15	10.384	+23 23 32.5	– 4.21	15 45.87	8.66	–2 26.67	–0.527	18 14 31.47
27	Th	6 21 7.30	10.378	23 21 39.1	5.24	15 45.83	8.66	2 39.26	0.522	18 18 28.03
28	Fr	6 25 16.32	10.373	23 19 21.2	6.26	15 45.79	8.66	2 51.72	0.516	18 22 24.59
29	Sa	6 29 25.18	10.366	23 16 38.7	7.28	15 45.76	8.66	3 4.03	0.509	18 26 21.15
30	Su	6 33 33.87	10.358	23 13 31.7	8.30	15 45.73	8.66	3 16.16	0.502	18 30 17.71
July 1	Mo	6 37 42.37	10.350	+23 10 0.3	– 9.32	15 45.71	8.66	–3 28.10	–0.493	18 34 14.27
2	Tu	6 41 50.65	10.340	+23 6 4.5	–10.33	15 45.69	8.66	–3 39.82	–0.484	18 38 10.82

NOTE.—0h Greenwich Civil Time is twelve hours before Greenwich Mean Noon of the same date.

local time, we must first change the given local time into the corresponding Greenwich time.

Example: Find the right ascension of the sun for July 1, 1929, $3^h\ 35^m$ P.M. at a place whose longitude is 5^h west of Greenwich.

		h	m
a.	Local Civil Time ($12^h + 3^h\ 35^m$)...	15	35
	Longitude from Greenwich (to be added)	5	0
	Corresponding Greenwich Civil Time.............	20	35

b. This time is $3^h.417$ ($24^h - 20^h\ 35^m$) before Greenwich 0^h of July 2.

	h	m	s
Sun's R.A., July 2 (from Ephemeris)	6	41	50.65
Change in $-3^h.417 = (-3.417) \cdot (+10^s.340) =$			-35.33
Required R.A........................	6	41	15.32

SUN, 1929.

FOR WASHINGTON APPARENT NOON

Date.		Apparent Right Ascension.	Var. per Hour.	Apparent Declination.	Var. per Hour.	Equation of Time. Mean—App.	Var. per Hour.	Semi-diameter.	S. T. of Sem. Pass Merid.	Sidereal Time of 0^h Civil Time.
		h m s	s	° ′ ″	″	m s	s	′ ″	m s	h m s
May	17	3 35 52.04	9.921	+19 19 48.3	+33.81	−3 45.41	+0.065	15 50.50	1 7.34	15 37 39.80
	18	3 39 50.43	9.944	19 33 9.9	32.98	3 43.58	0.088	15 50.31	1 7.42	15 41 36.35
	19	3 43 49.36	9.967	19 46 11.5	32.15	3 41.22	0.110	15 50.13	1 7.49	15 45 32.91
	20	3 47 48.83	9.989	19 58 52.8	31.30	3 38.33	0.132	15 49.95	1 7.57	15 49 29.47
	21	3 51 48.83	10.011	20 11 13.7	30.44	3 34.89	0.154	15 49.77	1 7.65	15 53 26.03
	22	3 55 49.36	10.033	+20 23 13.9	+29.57	−3 30.92	+0.176	15 49.60	1 7.72	15 57 22.58
	23	3 59 50.42	10.055	20 34 53.2	28.70	3 26.43	0.198	15 49.43	1 7.79	16 1 19.14
	24	4 3 52.00	10.076	20 46 11.4	27.81	3 21.43	0.219	15 49.26	1 7.87	16 5 15.70
	25	4 7 54.09	10 097	20 57 8.2	26.92	3 15.91	0 240	15 49.09	1 7.94	16 9 12.25
	26	4 11 56.69	10.118	21 7 43.4	26.01	3 9.88	0.261	15 48.92	1 8.00	16 13 8.81
	27	4 15 59.78	10.139	+21 17 56.8	+25.10	−3 3 36	+0.282	15 48.76	1 8.07	16 17 5.37
	28	4 20 3.36	10.159	21 27 48.2	24.18	2 56.36	0.302	15 48.60	1 8.14	16 21 1.93
	29	4 24 7.42	10.179	21 37 17.4	23.25	2 48.88	0.321	15 48.44	1 8.20	16 24 58.48
	30	4 28 11.95	10.198	21 46 24.2	22.31	2 40.94	0.340	15 48.28	1 8.26	16 28 55.04
	31	4 32 16.92	10.217	21 55 8.4	21.37	2 32.55	0.359	15 48.13	1 8.32	16 32 51.60
June	1	4 36 22.33	10.235	+22 3 29.8	+20.42	−2 23.72	+0.377	15 47.99	1 8.38	16 36 48.16
	2	4 40 28.17	10.252	22 11 28.3	19.45	2 14.46	0.394	15 47.85	1 8.44	16 40 44.72
	3	4 44 34.41	10.268	22 19 3.6	18.48	2 4.80	0.410	15 47.71	1 8.49	16 44 41.27
	4	4 48 41.03	10.283	22 26 15.5	17.51	1 54.76	0.426	15 47.58	1 8.54	16 48 37.83
	5	4 52 48.02	10.298	22 33 4.0	16.53	1 44.35	0.441	15 47.45	1 8.58	16 52 34.39
	6	4 56 55.36	10.312	+22 39 28.7	+15.54	−1 33.61	+0.454	15 47.33	1 8.63	16 56 30.95
	7	5 1 3.01	10.325	22 45 29.5	14.54	1 22.55	0.467	15 47.21	1 8.67	17 0 27.51
	8	5 5 10.95	10.336	22 51 6.3	13.53	1 11.20	0.478	15 47.10	1 8.71	17 4 24.06
	9	5 9 19.15	10.347	22 56 19.1	12.52	0 59.58	0.489	15 46.99	1 8.75	17 8 20.62
	10	5 13 27.60	10.357	23 1 7.6	11.51	0 47.73	0.498	15 46.89	1 8.78	17 12 17.18

Thus far in securing data for the sun, we have referred only to the meridian of Greenwich. Part II of the American Ephemeris gives similar data for the meridian of Washington, for Washington Apparent Noon (see page 145). The tables are especially convenient when it is required to obtain data for a given apparent time. The longitude of Washington is $5^h\ 8^m\ 15^s.78$ west of Greenwich.

APPARENT PLACES OF STARS, 1929.

CIRCUMPOLAR STARS.

FOR THE UPPER TRANSIT AT WASHINGTON.

43 H. Cephei. Mag. 4.52			α Ursæ Minoris. (*Polaris.*) Mag. 2.12			4 G. Octantis. Mag. 5.63			Groombridge 750. Mag. 6.70			Groombridge 944. Mag. 6.41		
Wash. Civil Time.	Right Ascension.	Declination.	Wash. Civil Time.	Right Ascension.	Declination.	Wash. Civil Time.	Right Ascension.	Declination.	Wash. Civil Time.	Right Ascension.	Declination.	Wash. Civil Time.	Right Ascension.	Declination.
	h m	° ′		h m	° ′		h m	° ′		h m	° ′		h m	°
June	0 58	+85 52	June	1 35	+88 55	June	1 41	−85 7	June	4 13	+85 21	June	5 38	+85 9
	s	″		s	″		s	″		s	″		s	″
0.3	28.24	24.94	0.4	19.04	14.18	0.4	11.29	24.97	0.5	19.36	62.62	0.5	45.31	65.00
1.3	28.47	24.81	1.4	19.81	14.00	1.4	11.48	24.69	1.5	19.41	62.36	1.5	45.28	64.73
2.3	28.71	24.66	2.4	20.66	13.81	2.4	11.63	24.43	2.5	19.46	62.07	2.5	45.24	64.44
3.3	28.99	24.52	3.4	21.60	13.62	3.4	11.79	24.17	3.5	19.54	61.78	3.5	45.23	64.12
4.3	29.30	24.41	4.4	22.63	13.43	4.4	11.93	23.91	4.5	19.63	61.46	4.5	45.22	63.80
5.3	29.61	24.30	5.4	23.74	13.28	5.4	12.07	23.65	5.5	19.76	61.14	5.5	45.24	63.46
6.3	29.94	24.21	6.4	24.91	13.13	6.4	12.20	23.38	6.5	19.91	60.84	6.5	45.28	63.11
7.3	30.27	24.16	7.4	26.08	13.01	7.4	12.33	23.08	7.5	20.08	60.55	7.5	45.34	62.79
8.3	30.59	24.13	8.4	27.21	12.92	8.4	12.48	22.76	8.5	20.25	60.30	8.5	45.43	62.50
9.3	30.89	24.12	9.4	28.28	12.86	9.4	12.66	22.44	9.5	20.43	60.06	9.5	45.51	62.21
10.3	31.18	24.11	10.3	29.27	12.80	10.4	12.86	22.12	10.5	20.57	59.86	10.5	45.58	61.97
11.3	31.44	24.08	11.3	30.18	12.72	11.3	13.08	21.81	11.5	20.71	59.66	11.5	45.64	61.72
12.3	31.68	24.04	12.3	31.05	12.63	12.3	13.30	21.53	12.5	20.83	59.43	12.5	45.69	61.48
13.3	31.92	23.99	13.3	31.92	12.53	13.3	13.52	21.27	13.4	20.93	59.21	13.5	45.72	61.22
14.3	32.19	23.93	14.3	32.83	12.40	14.3	13.73	21.03	14.4	21.04	58.97	14.5	45.73	60.94
15.3	32.46	23.86	15.3	33.80	12.29	15.3	13.93	20.83	15.4	21.15	58.70	15.5	45.76	60.65

35. To Obtain from the Ephemeris the Apparent Place, at a Given Instant, of a Circumpolar Star.—The apparent places of 17 northern and 18 southern circumpolar stars are given in order of their right ascensions in Part II of the Ephemeris for every day in the year, for the time of their upper transit at Washington. To obtain their apparent places at any other time, a *direct interpolation is necessary.*

Example: Compute the apparent declination of Polaris for June 3, 1929, 9^h 45^m P.M. at a place whose longitude is 8^h west of Greenwich.

	h	m	s
Given longitude..........................	8		
Longitude of Washington..................	5	8	15.78
Difference in longitude or time	2	51	44 22
Given civil time ($12^h + 9^h\ 45^m$)	21	45	0 00
Corresponding Washington Civil Time......	24	36	44 22

This is equivalent to June 4, 0^h 36^m $44^s.22$ or approximately, June $4^d.0$. From the Ephemeris (see page 46, of this book), we have:

	°	′	″
June $3^d.4$, declination of Polaris	88	55	13.62
June 4.4, declination of Polaris	88	55	13 43
Variation in one day.			0 19
Variation in 0.6 day = (0.19) · (0.6).......			0 11
Hence, required declination...	88	55	13.51

Observe:

a. Our first task was to change the given time into the corresponding Washington time.

b. June $3^d.4$ means that on June 3 at approximately 10^h ($0.4 \cdot 24^h$) Washington Civil Time, Polaris will cross the meridian of Washington.

36. To Obtain from the Ephemeris the Apparent Place, at a Given Instant, of the Ten-day Stars.—The apparent places of 790 stars are given in order of their right ascension in Part II of the Ephemeris for every 10 days throughout the year, for the times of their upper transits at Washington. In the first column (see page 48 of this book) of each page is given the day and tenths of day of the approximate Washington Civil Time of their upper transits. In the computation of the places for these stars, the short-period terms of the nutation were not included. If great exactness is required, these terms must be included.

Approximate apparent places of 10-*day stars* (correct to about a tenth of a second in R.A. and one-tenth of a second in declination) may be obtained from the tabulated values by direct interpolation.

APPARENT PLACES OF STARS, 1929.

FOR THE UPPER TRANSIT AT WASHINGTON.

Washington Civil Time.	α Virginis. (Spica.) Mag. 1.21 Right Ascension.		α Virginis. Declination.		9 B. Ursæ Minoris. Mag. 6.07 Right Ascension.		9 B. Ursæ Minoris. Declination.		70 Virginis. Mag. 5.16 Right Ascension.		70 Virginis. Declination.		ζ Virginis. Mag. 3.44 Right Ascension.		ζ Virginis. Declination.	
	h m		° ′		h m		° ′		h m		° ′		h m		° ′	
	13 21		−10 47		13 24		+72 44		13 24		+14 9		13 31		− 0 13	
	s		″		s		″		s		″		s		″	
Jan. 1.3	25.763	343	22.39	209	18.87	84	79.72	139	56.335	342	24.20	219	3.212	336	58.44	213
11.3	26.106	336	24.48	210	19.71	85	78.33	74	56.677	337	22.01	191	3.548	332	60.57	204
21.2	26.442	320	26.58	204	20.56	84	77.59	7	57.014	325	20.10	160	3.880	319	62.61	187
31.2	26.762	295	28.62	193	21.40	79	77.52	59	57.339	301	18.50	124	4.199	296	64.48	163
Feb. 10.2	27.057	266	30.55	176	22.19	72	78.11	122	57.640	270	17.26	84	4.495	269	66.11	139
20.1	27.323	234	32.31	156	22.91	61	79.33	178	57.910	239	16.42	46	4.764	238	67.50	109
Mar. 2.1	27.557	197	33.87	135	23.52	51	81.11	225	58.149	202	15.96	8	5.002	202	68.59	80
12.1	27.754	161	35.22	112	24.03	38	83.36	263	58.351	163	15.88	26	5.204	167	69.39	53
22.1	27.915	125	36.34	89	24.41	25	85.99	288	58.514	126	16.14	56	5.371	132	69.92	25
Apr. 1.0	28.040	94	37.23	67	24.66	11	88.87	302	58.640	91	16.70	80	5.503	100	70.17	3
11.0	28.134	62	37.90	47	24.77	1	91.89	304	58.731	57	17.50	99	5.603	68	70.20	18
	12		12		13		13		13		13		14		14	
20.9	28.196	33	38.37	28	24.76	14	94.93	293	58.788	29	18.49	112	5.671	41	70.02	34
30.9	28.229	9	38.65	13	24.62	25	97.86	273	58.817	1	19.61	118	5.712	14	69.68	47
May 10.9	28.238	15	38.78	1	24.37	36	100.59	241	58.816	23	20.79	119	5.726	9	69.21	56
20.9	28.223	36	38.77	14	24.01	44	103.00	204	58.793	45	21.98	114	5.717	32	68.65	61
30.9	28.187	54	38.63	26	23.57	50	105.04	160	58.748	64	23.12	107	5.685	50	68.04	64
June 9.8	28.133	72	38.37	35	23.07	55	106.64	111	58.684	81	24.19	96	5.635	68	67.40	65
19.8	28.061	86	38.02	42	22.52	60	107.75	59	58.603	91	25.15	80	5.567	81	66.75	63
29.8	27.975	97	37.60	49	21.92	61	108.34	5	58.512	104	25.95	63	5.486	94	66.12	59
July 9.8	27.878	105	37.11	54	21.31	62	108.39	50	58.408	112	26.58	46	5.392	104	65.53	54
19.7	27.773	110	36.57	58	20.69	61	107.89	102	58.296	115	27.04	24	5.288	109	64.99	46
29.7	27.663	111	35.99	60	20.08	58	106.87	153	58.181	113	27.28	2	5.179	110	64.53	38
Aug. 8.7	27.552	105	35.39	58	19.50	54	105.34	202	58.068	110	27.30	22	5.069	106	64.15	26
18.7	27.447	93	34.81	54	18.96	49	103.32	246	57.958	97	27.08	44	4.963	97	63.89	15
28.6	27.354	75	34.27	47	18.47	41	100.86	285	57.861	82	26.64	69	4.866	80	63.74	1
Sept. 7.6	27.279	52	33.80	36	18.06	34	98.01	322	57.779	56	25.95	94	4.786	59	63.75	18
17.6	27.227	19	33.44	20	17.72	25	94.79	350	57.723	27	25.01	119	4.727	28	63.93	38
27.5	27.208	18	33.24	3	17.47	15	91.29	372	57.696	9	23.82	148	4.699	7	64.31	60
Oct. 7.5	27.226	60	33.21	20	17.32	3	87.57	387	57.705	50	22.34	170	4.706	47	64.91	83
17.5	27.286	107	33.41	45	17.29	10	83.70	393	57.755	95	20.64	194	4.753	93	65.74	110
27.5	27.393	154	33.86	74	17.39	22	79.77	391	57.850	141	18.70	215	4.846	139	66.84	133
Nov. 6.4	27.547	201	34.60	101	17.61	35	75.86	381	57.991	189	16.55	235	4.985	186	68.17	158
16.4	27.748	245	35.61	130	17.96	47	72.05	358	58.180	232	14.20	247	5.171	230	69.75	179
26.4	27.993	282	36.91	155	18.43	59	68.47	328	58.412	273	11.73	254	5.401	269	71.54	198
Dec. 6.4	28.275	314	38.46	177	19.02	68	65.19	287	58.685	304	9.19	254	5.670	301	73.52	212
16.3	28.589	333	40.23	194	19.70	76	62.32	235	58.989	325	6.65	247	5.971	322	75.64	218
26.3	28.922	344	42.17	205	20.46	82	59.97	180	59.314	344	4.18	232	6.293	335	77.82	217
36.3	29.266		44.22		21.28		58.17		59.658		1.86		6.629		79.99	
Mean Place	26.946		28.35		19.316		95.69		57.363		26.96		4.371		60.44	
Sec δ, Tan δ	1.018		−0.191		3.374		+3.222		1.031		+0.252		1.000		−0.004	
$D_\psi\alpha$, $D_\omega\alpha$	+0.063		−0.012		+0.030		+0.200		+0.059		+0.016		+0.061		0.000	
$D_\psi\delta$, $D_\omega\delta$	−0.37		−0.35		−0.37		−0.36		−0.37		−0.36		−0.37		−0.39	

NOTE.—0h Washington Civil Time is twelve hours before Washington Mean Noon of the same date.

Example: Obtain the right ascension of Spica, Jan. 6, 1929, for the upper transit at Denver (long. $6^h\ 59^m\ 47^s.72$ W).

From the Ephemeris we have,

Date	Right ascension		Declination	
d	h m s		° ′ ″	
Jan. 1.3	13 21 25 763		−10 47 22 39	
		0 343		−2.09
Jan. 11.3	13 21 26.106		−10 47 24.48	

The transit at Washington will occur throughout the interval of 10 days, 0.3 of a day after Washington 0^h Civil Time. The transit at Denver will occur throughout the interval of 10 days, about 2^h ($6^h\ 59^m\ 47^s.72$ − $5^h\ 8^m\ 15^s.78$) later, or 0.4 $\left(0.3 + \frac{2}{24}\right)$ of a day after Washington 0^h Civil Time. That is, the required transit will occur on Jan. $6^d.4$ Washington Civil Time, hence the coordinates of Spica will be:

R.A. for Jan. $6^d.4 = 13^h\ 21^m\ 25^s.76 + 5.1 \cdot (0^s.034) = 13^h\ 21^m\ 25^s.9$.
Decl. for Jan. $6.4 = -10°\ 47'\ 22''.39 + 5.1 \cdot (-0''.209) = -10°\ 47'\ 23''.5$.

******To Obtain from Ephemeris the Apparent Place of Ten-day Stars.*—In the above example the short-period terms for nutation were not included. These terms attain two maxima and two minima during a month. At maximum and minimum they may amount in right ascension to $\pm(0^s.020 + 0^s.008 \tan\delta)$ and in declination to $\pm 0''.13$.

To compute the apparent place of a star at any instant we must first interpolate for the given date from the 10-day Ephemeris, and second apply to this the correction $\Delta\alpha$ for the effect of the short-period terms in right ascension and $\Delta\delta$ for the corresponding effect in declination. From the American Ephemeris:[1]

$$\left.\begin{aligned}\Delta\alpha &= D_\psi\alpha \cdot \delta''\psi + D_\omega\alpha \cdot \delta''\omega.\\ \Delta\delta &= D_\psi\delta \cdot \delta''\psi + D_\omega\delta \cdot \delta''\omega\end{aligned}\right\} \qquad (28)$$

$D_\psi\alpha$, $D_\omega\alpha$, $D_\psi\delta$, and $D_\omega\delta$ are the coefficients of the short-period terms of the nutation and are given for each star at the foot of the page of Apparent Places (see foot of page 48 in this book). $\delta''\psi$ and $\delta''\omega$ are the terms of short period in nutation and are given in the Ephemeris under that title.[2]

[1] A.E., p. 201, 1929.
[2] A.E., pp. 215 and 216, 1929.

Example: Compute the apparent place of Spica, Jan. 6, 1929, for the upper transit at Denver (long. $6^h\ 59^m\ 47^s72$ W). We interpolate first, for the 5.1 days [as before].

$$\text{Jan. } 6^d4 \quad \alpha = 13^h\ 21^m\ 25^s\!.763 + 5.1 \cdot (0^s\!.034) = 13^h\ 21^m\ 25^s\!.936.$$
$$\delta = -10°\ 47'\ 22''39 + 5.1 \cdot (-0''209) = -10°\ 47'\ 23''46.$$

From the foot of the same page of the Ephemeris (see page 48 of this book), we obtain,

$$D_\psi\alpha = 0.063 \quad D_\omega\alpha = -0.012$$
$$D_\psi\delta = -0.37 \quad D_\omega\delta = -0.35.$$

We obtain from the Ephemeris for the nutation terms of short period the following:

Date	$\delta''\psi$	$\delta''\omega$
Jan. 6	−0.17	−0.01
Jan. 7	−0.16	−0.05

Hence, for

Jan. 6.4	−0.17	−0.03

Equation (28) gives

$$\Delta\alpha = 0.063 \cdot (-0.17) + (-0.012) \cdot (-0.03) = -0^s\!.011.$$
$$\Delta\delta = -0.37 \cdot (-0.17) + (-0.35) \cdot (-0.03) = +0''\!.07.$$

Therefore,

$$\alpha = 13^h\ 21^m\ 59^s\!936 + \Delta\alpha = 13^h\ 21^m\ 59^s\!.925.$$
$$\delta = -10°\ 47'\ 23''\!.46 + \Delta\delta = -10°\ 47'\ 23''\!.39.$$

37. Double Interpolation.—The last part of the American Ephemeris contains many useful tables to facilitate the determination of approximate latitude and azimuth from observations on Polaris. These tables present the tabular values in terms of two variables, and to obtain the value required a double interpolation is necessary. The tabular intervals are small, hence, a straight-line interpolation is made for each variable beginning with the nearest tabulated value.

Example: Suppose it is required to find the azimuth of Polaris in 1930, when its hour angle is $8^h\ 34^m$, at a latitude of 38° 40′ N. Table IV of the 1930 Ephemeris gives

AZIMUTH OF POLARIS

H.A. \ Lat.	38°	40°
h m	° ′	° ′
8 30	1 3.9	1 5.7
8 40	1 1.7	1 3.4

For the hour angle of $8^h\ 34^m$ and lat. of 38° N the azimuth $= 1°\ 3'.0$
For the hour angle of $8^h\ 34^m$ and lat. of 40° N the azimuth $= 1°\ 4'.8$

Hence
For the hour angle of $8^h\ 34^m$ and lat. of 38° 40′ N the azimuth $= 1°\ 3'.6$.

The interpolation may also be effected as follows. The table shows that the azimuth *decreases* with hour angle and *increases* with latitude. The decrease due to hour angle is $\frac{4}{10} \cdot 2'.2 = 0'.88$. The increase due to latitude is $\frac{40}{120} \cdot 1'.8 = 0'.6$. Hence the required value is,

$$1°\ 3'.9 - 0.9 + 0'6 = 1°\ 3'.6$$

and the star is 1° 3′.6 west of north. This method is well adapted to cases where more than two variables are involved.

38. Star Catalogues.—For ordinary observations the list of stars given in the Ephemeris is usually sufficient, but occasionally it is desirable to employ stars not included in this list. If we observe such stars, *star catalogues* must be consulted to obtain their coordinates. These catalogues contain many thousands of stars whose positions have been accurately observed and their mean places computed for the beginning of a stated fictitious year. The beginning of the fictitious year differs from the beginning of the ordinary year by a fraction of a day, and it is defined as the instant at which the celestial longitude of the mean sun is 280° (celestial longitude in the system of coordinates where the ecliptic is the fundamental circle corresponds to right ascension in the equator system).

Stars in these catalogues are arranged according to their right ascensions. One of the most important catalogues is Boss's "Preliminary General Catalogue." It is computed for the epoch of 1900, and contains 6,188 stars.

***39. Reduction of Mean Place of a Star from One Epoch to Another.**—Usually catalogues give the mean place for the beginning of the fictitious year of the catalogue. To change this to the mean place of some other year, with a high degree of accuracy, is a long and involved piece of work, and the method is not to be given here. However, for the purposes of the astronomer in general, a sufficient degree of accuracy may be obtained by using the reduction formulas given in modern catalogues.

The work of reduction is expressed in the form of two formulas, one for right ascension and the other for declination. These are usually explained at the beginning of the catalogue. They may be given in the form:

$$\alpha_0 \text{ (epoch } T) = \alpha_0' \text{ (epoch } T_0) + \text{(annual variation)} \cdot t + (\tfrac{1}{200} \text{ secular var.}) \cdot t^2.$$
$$\delta_0 \text{ (epoch } T) = \delta_0' \text{ (epoch } T_0) + \text{(annual variation)} \cdot t + (\tfrac{1}{200} \text{ secular var.}) \cdot t^2.$$

Where α_0 and δ_0 are the mean coordinates of the star for the beginning of any year T, and α_0', and δ_0' are the coordinates of the star given in the catalogue of the year T_0. The annual and secular variations are given for each star in the catalogue. $T - T_0 = t$, number of years since the epoch of the catalogue.

In some catalogues, *e.g.*, Boss's "Preliminary General Catalogue," the annual variation includes μ and μ', the annual proper motions in right ascension and declination, respectively. If they are not included add μt to α_0 and $\mu' t$ to δ_0.

Example: Find the mean place of B.3080 for the epoch of 1929. From Boss's General Catalogue, we obtain:

	Magn.	R A , 1900	An var	Sec var	Decl., 1900	An. var.	Sec. var
B.3080	5.3	$11^h\ 36^m\ 44^s.294$	$+2^s.9877$	$+0^s.0181$	$-31°\ 56'\ 38''.71$	$-20''.010$	$-0''.036$

and from the above formula, we have:

$$\alpha_0(\text{Jan. } 0^d.6,\ 1929) = 11^h\ 36^m\ 44^s.294 + 2^s.9877 \cdot 29 + \tfrac{1}{200}(0^s.0181) \cdot (29)^2 = 11^h\ 38^m\ 11^s.013.$$
$$\delta_0(\text{Jan. } 0^d.6,\ 1929) = -31°\ 56'\ 38''.71 + (-20''.010) \cdot 29 + \tfrac{1}{200}(-0''.036) \cdot (29)^2 = -32°\ 6'\ 19''.15.$$

***40. Reduction of Mean Place at the Beginning of a Year, to the Apparent Place at Any Given Instant.**—Having the mean place (Art. 39) for the beginning of the desired year, to obtain the apparent place for the instant required, we must correct for precession and proper motion for the fraction of the year and likewise for nutation and annual aberration. The formulas for this reduction, together with explanations are given in the American Ephemeris, (see pages 200 and 201 of year 1930). There are two sets of such formulas given; the second set is a little more desirable when reductions are to be made for only a few stars. This set is:

$$\alpha = \alpha_0 + f + f' + \tau\mu + \tfrac{1}{15} g \sin (G + \alpha_0) \tan \delta_0 + \tfrac{1}{15} h \sin (H + \alpha_0) \sec \delta_0 \qquad \text{(in time).}$$
$$\delta = \delta_0 + \tau\mu' + g \cos (G + \alpha_0) + h \cos (H + \alpha_0) \sin \delta_0 + i \cos \delta_0 \qquad \text{(in arc).}$$

The significance of the symbols used, and an example worked in detail, may be found on page 761 (year 1930) of the American Ephemeris.

Example: Star: B.5283 Date: July 30, 1930

	°	′		h	m	s
G	317	46.2	Cleveland Sidereal Time.	20	31	39
α_0	307	54.0	Correction to Washington	0	18	
H	146	0.4	Washington Sidereal Time.	20	49	39
$(G + \alpha_0)$	625	40 2				
$(H + \alpha_0)$	453	54 4				

$$\mu = +0^s.0028 \quad \mu' = -0''.003 \quad \tau = 0.5754$$

		log g	1.0784
log $\frac{1}{15}$	8.8239 − 10	log cos $(G + \alpha_0)$	8.8780 − 10 (n)
log g	1.0784		
log sin $(G + \alpha_0)$	9.9988 − 10 (n)		9.9564 − 10 (n)
log tan δ_0	0.0220		$-0''.905$
	9.9231 − 10 (n)	log h	1.2987
	$-0^s.838$	log cos $(H + \alpha_0)$	8.8333 − 10 (n)
		log sin δ_0	9.8602 − 10
log $\frac{1}{15}$	8.8239 − 10		
log h	1.2987		9.9922 − 10 (n)
log sin $(H + \alpha_0)$	9.9990 − 10		$-0''.982$
log sec δ_0	0 1618		
		log i	0 6834
	10.2834 − 10	log cos δ_0	9.8382 − 10
	$+1^s.920$		
			10.5216 − 10
			$+3''324$

	h	m	s		°	′	″
α_0	20	31	37.393	δ_0	46	27	9.929
$\tau\mu$			+0.002	$\tau\mu'$			−0 002
f			+1.356				−0.905
f'			+0.003				−0.982
			−0.838				+3.324
			+1.920				
				$\delta =$	46	27	11.36
$\alpha =$	20	31	39.836				

Exercises

1. The angle that the line joining the earth and a star makes with the direction of the earth's motion is 90°. The orbital velocity of the earth is

18.5 miles per second, the velocity of light is 186,300 m.p.s. Find the angular displacement of the star due to annual aberration.

2. The following stars were observed as they appeared to be crossing the upper meridian of Cleveland (lat. 41° 32′ N):

	Declination
α Andromedæ	+28° 42′
β Ceti	−18° 22′
38 Cassiopeiæ..........	+69° 54′

Find how late they crossed the meridian on account of the diurnal aberration.

3. How many years would it take the vernal equinox to move 10°?

4. Obtain the apparent R.A., declination of the sun, and equation of time, for June 11, 1929, 7^h G.C.T.

5. Obtain the equation of time for June 1, 1929, 4^h 30^m P.M. Local Civil Time at a place whose longitude is 6^h 30^m west of Greenwich.

6. Obtain the apparent declination of the sun for Dec. 11, 1929, 9^h Washington Apparent Time.

7. Obtain the apparent R.A. and declination of Groombridge 750 (see page 46 of this book) for June 6, 1929, 5^h 15^m A.M. at a place whose longitude is 2^h 10^m east of Greenwich.

8. Obtain the apparent place (give results to tenths of seconds) of 70 Virginis (see page 48 of this book), Nov. 7, 1929, for the upper transit at St. Louis (long. 6^h 0^m $49\overset{s}{.}26$ W).

***9.** Obtain the apparent place of 70 Virginis, Nov. 7, 1929, for the upper transit at St. Louis (long. 6^h 0^m $49\overset{s}{.}26$ W). Include in your result the short-period terms of nutation.

10. The following data were taken from the American Ephemeris:

Azimuth of Polaris, 1930

H.A. \ Lat.	+48°	+50°
h m	° ′	° ′
7 0	1 32.0	1 35 7
7 10	1 30 8	1 34 4

By means of double interpolation, compute the azimuth of Polaris when its hour angle is 7^h 8^m 15^s, at the latitude of 48° 51′ N.

***11.** From Boss's "Preliminary General Catalogue," we obtain the mean place of υ Leonis for the epoch of 1900. Find its mean place for the epoch 1930.

Coordinates		An. var.	Sec. var.
R. A., 1900......	11^h 31^m $49\overset{s}{.}716$	$+3\overset{s}{.}0716$	$+0\overset{s}{.}0004$
Decl., 1900.......	$-0°$ $16'$ $18\overset{''}{.}10$	$-19\overset{''}{.}861$	$-0\overset{''}{.}046$

***12.** Obtain the result of Exercise 9 from the mean place of 70 Virginis, given in the American Ephemeris.

CHAPTER V

CONVERSION OF TIME

We have seen in Chap. III, that the common mode of time reckoning in everyday life is Standard Time, and that in astronomy, we deal largely with three other systems of time reckoning which, by the nature of their definitions, are *local times*. These are:

1. Apparent Solar Time (T_a).
2. Mean Solar or Civil Time (T).
3. Sidereal Time (θ).

In this chapter we are concerned with two main problems:

a. Given the local time of a certain instant at one place in one system of reckoning, to find the corresponding local time of the same instant for another place.

b. Given the local time of a certain instant in one system of reckoning, to find for the *same place* the local time of the same instant in another system of reckoning.

41. To Change Local Time of a Place of Given Longitude, to Greenwich Time and Vice Versa.—The relation between the local times of two places and their respective longitudes has been explained in Art. 27. The method of converting time where one of the two places is Greenwich will be illustrated by the following examples:

1. What is the Greenwich Civil Time, when the civil time of a place of longitude $5^h\ 10^m\ 15^s$ W is $7^h\ 15^m\ 12^s$?

	h	m	s
Given civil time..............................	7	15	12
Longitude west of Greenwich................	5	10	15
Corresponding civil time at Greenwich	12	25	27

2. Change April 12, $22^h\ 5^m\ 51^s$ civil time of a place of longitude $6^h\ 25^m\ 13^s$ W, into corresponding Greenwich Civil Time.

	h	m	s
Given civil time, April 12..	22	5	51
Longitude west of Greenwich	6	25	13
Corresponding Greenwich time	28	31	4

or

April 13	4	31	4

3. The Greenwich Sidereal Time of an instant is $17^h\ 41^m\ 52^s$. Find the corresponding time of a place of longitude $4^h\ 38^m\ 12^s$ W.

	h	m	s
Given Greenwich Sidereal Time	17	41	52
Longitude west of Greenwich	4	38	12
Required sidereal time	13	3	40

Observe that the difference in time between Greenwich and the given place is equal to the longitude of the place west of Greenwich, and that the more easterly place has the later time.

42. To Change Standard to Civil Time and Vice Versa. (See Art. 28.)

Example.

1. What is the civil time of a place of longitude $5^h\ 26^m\ 16^s$ W, when the Eastern Standard Time is $7^h\ 12^m\ 15^s$ A.M.?

	h	m	s
Given Eastern Standard Time	7	12	15
Longitude *west* of 75° meridian		26	16
Civil time	6	45	59

2. What is the Pacific Standard Time of $15^h\ 12^m\ 19^s$ civil time at a place of longitude $7^h\ 58^m\ 42^s$ W?

	h	m	s
Given civil time	15	12	19
Longitude *east* of the 120° meridian		1	18
Pacific Standard Time	15	11	1
=	3	11	1 P.M.

43. To Change Mean Time into Apparent Time ($T \rightarrow T_a$).— As has been explained in Art. 24, apparent time and mean time are connected by the equation,

$$T_a - T = E. \tag{29}$$

The American Ephemeris gives the equation of time at 0^h G.C.T. for every day in the year; to obtain its value for any other instant an interpolation (Art. 34) is necessary.

Example.

Find the apparent time of $6^h\ 30^m\ 0^s.0$, Greenwich Mean Time, May 20, 1929.

	h	m	s
Equation of time of 0^h G.C.T., May 20, 1929[1]		+ 3	40.4
Change of E in $6^h\ 30^m$ = $(6.5) \cdot (-0^s.116)$ =		−	0.8
Equation of time at $6^h\ 30^m$, (E)		+ 3	39.6
Given Greenwich time, (T)	6	30	0.0
Apparent time $(T_a = E + T)$	6	33	39 6

To change mean time of any place into apparent time, we must first convert the given mean time into the corresponding Greenwich Mean Time (Art. 41) for the purpose of obtaining the equation of time for the given instant. Having the equation of time, substitute in Eq. (29) as before.

Example.

The Cleveland Civil Time is 1929, July $1^d\ 14^h\ 11^m\ 10^s.0$. What is the apparent time?

	h	m	s
a. Cleveland Civil Time, 1929, July 1	14	11	10.0
Longitude of Cleveland	+ 5	26	16.4
Greenwich Civil Time, July 1	19	37	26.4
b. Equation of time at 0^h G.C.T., July 2		−3	39.8
Change of equation of time in $4^h\ 22^m\ 33^s.6$ $(24^h - 19^h\ 37^m\ 26^s.4)$ before 0^h = $(-4.376) \cdot (-0^s.484)$ =		+	2.1
Equation of time at given instant, (E) =		−3	37.7
c. Cleveland Civil Time, 1929, July 1, (T) =	14	11	10.0
Apparent time $(T_a = E + T)$ =	14	7	32.3

Observe that there are essentially three steps in the conversion.

a. From the given civil time obtain the corresponding Greenwich Civil Time.

[1] See American Ephemeris; or, for this illustration, p. 44 of this book.

b. With this time secure the equation of time from the Ephemeris.

c. Add this equation of time, algebraically, to the *given* civil time.

44. To Change Apparent Time into Mean Time ($T_a \rightarrow T$). *First Method.*—Since the Ephemeris gives the equation of time at 0^h G.C.T., to secure the equation of time necessary for the conversion from apparent to mean time, we must have at least an approximation of the Greenwich Civil Time of the given instant. This may be obtained accurately enough for our purpose by applying to the given apparent time the equation of time for the nearest 0^h G.C.T. Having this approximate Greenwich Civil Time, the correct equation of time may be obtained and, hence, from $T_a - T = E$, the required mean time.

Example: The Cleveland Apparent Time is, 1929, May 25, $5^h\ 30^m\ 0^s.0$. What is the mean time?

	h	m	s	
Cleveland Apparent Time, 1929, May 25 .	5	30	0 0	(1)
Longitude of Cleveland	5	26	16 4	(2)
Greenwich Apparent Time, May 25, (1) + (2)	10	56	16.4	(3)
Equation of time, May 25, at 0^h G.C.T		+3	19.9	(4)
Approximate Greenwich Civil Time ($T = T_a - E$), (3) − (4).	10	52	56 5	(5)
Change of equation of time in $10^h\ 52^m\ 56^s5$ = $10.882 \cdot (-0^s225)$ =			−2 5	(6)
Corrected equation of time at the given instant, (4) + (6).		+3	17.4	(7)
Required Cleveland Civil Time, (1) − (7). . . .	5	26	42.6	

Second Method.—The Ephemeris also gives the equation of time for Washington apparent noon (Part II; see page 45 of this book); therefore, we may proceed as follows:

a. Change the given apparent time into that of Washington.

b. Inasmuch as the tabular values given in this table are for the instant of Washington apparent noon, obtain the interval of time before or after Washington apparent noon.

c. Modify the tabulated equation of time nearest to the given date for this interval. This will be the equation of time of the given instant. Using this method for the above example, we have:

	h	m	s	
Cleveland Apparent Time, 1929, May 25	5	30	0.0	(1)
Longitude of Cleveland minus longitude of Washington		18	0 6	(2)
Corresponding Washington app. time, (1) + (2)........................	5	48	0.6	(3)
Equation of time for Washington app. noon[1] May 25, 1929......		−3	15.9	(4)
Change in equation of time in $6^h\ 11^m\ 59^s.4$ = $(-6.200) \cdot (+0^s.240)$ =			−1.5	(5)
Correct equation of time for given instant, (4) + (5)		−3	17.4	(6)
Required Cleveland Civil Time, (1) + (6)..	5	26	42.6	

45. To Change Mean Time into Sidereal Time ($T \rightarrow \theta$). *First Method.*—We have seen in Art. 25 that the sidereal clock gains on the mean, $3^m\ 56^s.56$ in 24 mean hours and that this gain for any interval may be obtained from Eqs. (18) or (19) or from tables. (See Tables II and III of this book or II and III of the American Ephemeris.) It is therefore necessary in obtaining the reading of the sidereal clock for the instant when the mean clock reads a given time, to *know the readings of the two clocks at some one instant.* The American Ephemeris (pages 2 to 17) gives,[2] for every day in the year, the reading of the sidereal clock at Greenwich when the mean clock at Greenwich reads 0^h.

Example 1: When the Greenwich mean clock reads 10^h on May 17, 1929, find the reading of the sidereal clock.

	h	m	s
The instant the civil clock at Greenwich reads 0^h, on May 17, the sidereal clock reads	15	36	49.16
In 10 mean hours, the sidereal clock will gain on the mean clock (Table III).....		1	38.57
	+10		
The required sidereal time is............	25	38	27 73
or	1	38	27 73

If the given civil time is the local time of some place other than that at Greenwich, it is first changed into Greenwich time (Art. 41) and the problem is solved as before.

[1] In this part of the Ephemeris, the equation of time is defined by mean − apparent = equation of time.

[2] See last column, p. 44 of this book.

Example 2: The Chicago Civil Time on 1929, June 6, is $15^h\ 10^m\ 0^s.00$. What is the sidereal time?

	h	m	s	
Chicago Civil Time (T)...	15	10	0.00	*(1)*
Longitude of Chicago..	5	50	26.84	*(2)*
Corresponding Greenwich Civil Time, *(1)* + *(2)*..................	21	0	26.84	*(3)*
Sidereal time of 0^h G.C.T., June 6..	16	55	40.31	*(4)*
Gain of sidereal clock in $21^h\ 0^m\ 26^s.84$ (Table III).......................		3	27.06	*(5)*
Greenwich Sidereal Time, *(3)* + *(4)* + *(5)*...	37	59	34.21	
or	13	59	34.21	*(6)*
Longitude of Chicago....................	5	50	26.84	*(7)*
Required sidereal time, *(6)* − *(7)*, θ =	8	9	7.37	

Second Method.—The same result will be obtained if we first find the reading of the sidereal clock of the given place when the civil clock of this place reads 0^h. The civil clock at Greenwich reads λ hours when the civil clock of a place of longitude λ west reads 0^h. It is therefore necessary to obtain the reading of the sidereal clock of Greenwich when the civil clock of Greenwich reads λ^h and from it subtract the given longitude of the place. This will be the reading of the local sidereal clock when the mean local civil clock reads 0^h.

From the example given above:

	h	m	s
The sidereal time for June 6, 0^h G.C.T. is...	16	55	40.31
and $5^h\ 50^m\ 26^s.84$ (long. of Chicago) mean time interval later it will be.............	5	50	26.84
plus the reduction of this mean time interval into sidereal (Table III)................			57.57
Hence the reading of the sidereal clock of Greenwich, when the mean clock reads $5^h\ 50^m\ 26^s.84$ is.	22	47	4.72
Longitude of Chicago....	5	50	26.84
Or the reading of sidereal clock of Chicago when the mean clock reads 0^h is	16	56	37.88

The difference between the readings of the sidereal clocks of Chicago and Greenwich when their mean clocks read 0^h is

57^s57, which is evidently the quantity used in reducing 5^h 50^m 26^s84 mean time interval into sidereal. *Therefore, to obtain the sidereal time of a place of longitude λ^h west of Greenwich, when the local civil clock of the place reads 0^h, add to the sidereal time of Greenwich 0^h the quantity used in reducing λ^h mean time interval into sidereal.* This is computed once for all for a given longitude and, in fact, is printed in the Table of Observatories, in the American Ephemeris, for the principal observatories under the heading Reduction from Greenwich to local S.T. of 0^h. For longitudes east of Greenwich this quantity is subtracted as the sign of the table indicates.

The solution of the above example, according to this method, is:

	h	m	s	
a. Sidereal time of 0^h G.C.T., June 6..........	16	55	40.31	
Reduction for 5^h 50^m 26^s84 from Table III..			+57.57	
Sidereal time of June 6, 0^h Chicago Civil Time.	16	56	37.88	(*1*)
b. The given civil time (T)...	15	10	0.00	
To reduce the interval of 15^h 10^m into sidereal interval (Table III) by adding........		2	29.49	
We obtain.	15	12	29.49	(*2*)
c. Required sidereal time (θ) is (*1*) + (*2*)... ...	32	9	7.37	
or	8	9	7.37	

There are three steps in the above solution:

1. The reduction of the sidereal time of 0^h G.C.T. of the given date into local sidereal time of 0^h local civil time.

2. The change of the given civil time interval after 0^h into sidereal interval.

3. The addition of this interval to the local sidereal time of 0^h civil time.

46. To Change Sidereal Time into Mean Time ($\theta \rightarrow T$). *First Method.*—The process is exactly the reverse of that given for changing mean time into sidereal.

Example: On June 10, 1929, the sidereal clock of a place of longitude 60° W reads 18^h. Find the reading of the civil clock at that instant.

	h	m	s	
Given local sidereal time (θ)...............	18	0	0.00	
Given longitude west of Greenwich.........	4	0	0.00	
Corresponding Greenwich Sidereal Time, June 10.	22	0	0 00	(*1*)
Sidereal time of 0^h G.C.T., June 10...... ..	17	11	26.54	(*2*)
Sidereal time interval after 0^h G.C.T., (*1*) − (*2*)........................	4	48	33.46	(*3*)
Reduction to mean time interval, from Table II, for $4^h\ 48^m\ 33^s.46$...........			47 27	(*4*)
Greenwich Civil Time, (*3*) − (*4*)...	4	47	46.19	(*5*)
To obtain from this the required local civil time (T), subtract the longitude of the place	4	0	0.00	(*6*)
Required Civil Time, T (*5*) − (*6*)...........	0	47	46.19	

Second Method.—As in the second method of the converse problem (Art. 45), obtain for the given date the local sidereal time of 0^h local civil time. The difference between the given sidereal time and the sidereal time just obtained is the time interval after 0^h local civil time. This interval is expressed in sidereal units; when changed into mean time units by Table II, it will give the required mean time.

Example: On June 30, 1929, the sidereal clock of Cleveland reads $15^h\ 10^m\ 10^s.00$. What is the reading of the mean clock at that instant?

	h	m	s	
a. Sidereal time 0^h G.C.T., June 30	18	30	17.71	
Reduction for the longitude of Cleveland of $5^h\ 26^m\ 16^s.36$ from Table III.			53.60	
Sidereal time of 0^h Cleveland Civil Time, June 30.........................	18	31	11.31	(*1*)
b. The given sidereal time (θ)	15	10	10.00	(*2*)
To obtain the interval after 0^h C.C.T., subtract (*1*) from (*2*); to do so add to (*2*).	24			
	39	10	10.00	(*3*)
Sidereal interval after 0^h Cleveland Civil Time, (*3*) − (*1*)...........	20	38	58.69	
c. Reduction of $20^h\ 38^m\ 58^s.69$ to mean interval (Table II)................		−3	22.98	
T =	20	35	35.71	

47. Some Hints in Methods of Computation.—In performing numerical computations, it is well to have in mind the following:

1. Do all your work in ink.
2. Arrange a convenient outline of the computation to be performed, preceding each number to be shown by a statement or name.
3. Do not carry unnecessary decimals. Be guided in this matter by the accuracy required in the result and by the limit of accuracy of the tables you are using.
4. Secure all the numbers wanted from the same page of your tables at the same time.

Exercises

1. What is the reading of the civil clock at Greenwich, the instant, (*a*) the civil clock of a place of longitude $4^h\ 50^m$ W reads 3 P.M.? (*b*) the Eastern Standard Time is 6 A.M.?

2. What is the Central Standard Time the instant the Greenwich civil clock reads (*a*) 8^h? (*b*) 2^h?

3. Change 1929, June $25^d\ 5^h\ 0^m\ 0\overset{s}{.}0$ G.C.T. into Greenwich Apparent Time.

4. On May 21, 1929, at a place whose longitude is $7^h\ 35^m$ W, the civil clock reads $5^h\ 31^m\ 12\overset{s}{.}1$ P.M.; what is the corresponding apparent time?

5. Change 1929, June $2^d\ 16^h\ 0^m\ 0\overset{s}{.}0$ Washington Apparent Time, into Washington Civil Time.

6. On April 30, 1929, at a place whose longitude is 105° W the apparent time is $14^h\ 12^m\ 41\overset{s}{.}7$. Find the corresponding civil time. Check result by another method.

7. What is the civil time of apparent noon, on June 1, 1929, at a place of longitude $5^h\ 3^m\ 27\overset{s}{.}3$ W? What is the Eastern Standard Time at the same instant?

8. On May 30, 1929, the civil clock of Greenwich reads 7^h. What is the reading of the sidereal clock?

9. On July 2, 1929, the civil clock of Greenwich reads 21^h. What is the reading of the sidereal clock?

10. If, on May 20, 1929, the civil clock of a place of longitude 6^h W reads 0^h, what is the reading of its sidereal clock at the same instant?

11. Change 1929, June 22^d, 0^h civil time, into sidereal time, at a place of longitude 30° E.

12. The civil time of a place of longitude $5^h\ 30^m$ W is, on 1929, June 18^d, $12^h\ 5^m\ 39\overset{s}{.}72$. What is the sidereal time?

13. The Greenwich sidereal clock on June 1, 1929, reads $19^h\ 0^m\ 0\overset{s}{.}0$. What is the reading of the mean clock?

14. The sidereal clock of Cambridge, Mass. (long. $4^h\ 44^m\ 31\overset{s}{.}05$ W) on June 11, 1929, reads $21^h\ 20^m\ 51\overset{s}{.}73$. What is the reading of the civil clock?

15. On June 12, 1929, at a place whose longitude is $10^h\ 0^m\ 11\overset{s}{.}83$ W. the sidereal time is $6^h\ 53^m\ 12\overset{s}{.}81$. Find the civil time of the instant.

16. On May 20, 1929, the sidereal clock of Greenwich reads 0^h. What is the corresponding reading of the civil clock? What is the reading, at the same instant, of the civil clock at a place of longitude $4^h\ 50^m\ 10\overset{s}{.}71$ W?

CHAPTER VI

CORRECTIONS TO OBSERVATIONS

Practical astronomy deals essentially with the solution of the astronomical triangle. Observed data to be used in the solution of this triangle must be corrected for instrumental and other errors. In Chap. VII we shall consider the corrections due to the imperfect adjustments of instruments; and in this chapter, corrections due to other causes. The correction for aberration was considered in Chap. IV.

48. Refraction.—On account of refraction, a ray of light from a heavenly body passing through the atmosphere suffers a change

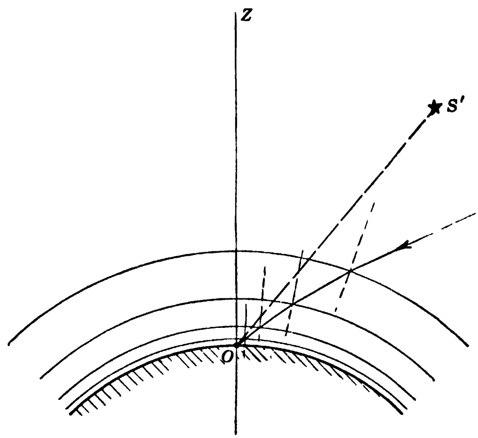

FIG. 27.—Earth's atmospheric refraction of light.

in direction. If we assume the atmosphere in horizontal layers of increasing density as we approach the surface of the earth, the light ray, as it passes from one layer to a denser one, will bend toward the *normal* to the interface of the two layers in the manner shown in Fig. 27. When the ray reaches the observer at O the object S will be seen in the direction from which its light enters the eye of the observer so that the heavenly body appears at S'. For this reason *all objects appear higher above the horizon* than they actually are. This displacement

is known as *astronomical refraction.* The angular amount of the displacement is called the *refraction correction* (r) and *must be added to the observed zenith distance.*

A simple and tolerably accurate formula may be developed by neglecting the earth's curvature and by assuming that all the refraction takes place at the upper surface of the atmosphere. Figure 28 shows the ray of light SA refracted at A toward the observer at O. The angle ZAS is the true zenith distance z

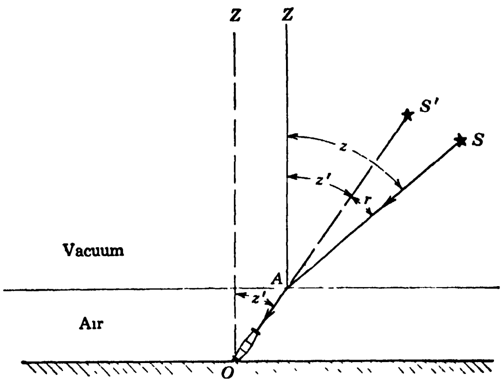

FIG. 28.—Refraction correction.

of the heavenly body and ZOS' or ZAS' the observed zenith distance z'. Hence,

$$z = z' + r. \tag{30}$$

The physical law of refraction from a vacuum to air may be expressed by the equation

$$\sin z = \mu \sin z', \tag{31}$$

where μ is the index of refraction. For air at zero degrees Centigrade and pressure of 76 cm. μ is 1.000294. Substituting Eq. (30) in Eq. (31) we have,

$$\sin (z' + r) = \mu \sin z'$$

or

$$\sin z' \cos r + \cos z' \sin r = \mu \sin z'.$$

Since r is very small (under average conditions not greater than $35'$ near the horizon) we may write 1 for $\cos r$, and r (in radians) for $\sin r$, hence the last equation becomes

$$\sin z' + r \cos z' = \mu \sin z'$$

or

$$r = (\mu - 1) \cdot \tan z'.$$

Substituting the value of μ and reducing r into seconds (1 radian = 206265″), we have

$$r = 0.000294 \cdot 206265'' \tan z'$$

$$\boxed{r = 60.''6 \tan z'} \tag{32}$$

The error in this formula of mean refraction becomes greater as the zenith distance increases and for 70° zenith distance is about 8″. Table IV gives a more accurate value of mean refraction. As the index of refraction μ depends upon the atmospheric temperature and pressure, a formula expressing the refraction correction in terms of these two quantities will yield more accurate results. For example, a closer approximation to the refraction correction is given by Comstock's empirical formula

$$r \text{ (in seconds of arc)} = \frac{983 \cdot b}{460 + t} \cdot \tan z' \tag{33}$$

where b is the barometric pressure in inches, t the temperature in degrees Fahrenheit, and z' the observed zenith distance. For zenith distances under 75° this formula will seldom be over 1″ in error.

49. Dip of the Horizon.—When the altitude of a heavenly body is observed at sea, its angular distance above the visible horizon or sea line is measured. But, owing to the curvature of the earth, the visible horizon is below the true horizon. Hence the angle between the visible horizon and the true horizon, known as the *dip of the horizon*, must be *subtracted* from the observed altitude of the heavenly body to obtain its true altitude. The magnitude of this correction depends upon the elevation of the observer above sea level. Let the observer at O (Fig. 29) be d ft. above sea level; h' be the observed altitude of the star S, and h its true altitude (refraction is neglected for the moment). If the earth is regarded as a sphere of radius R, the dip of the horizon D will equal the angle OCV. From triangle OCV, we have

$$\cos D = \frac{R}{R + d} = \frac{1}{1 + \frac{d}{R}} = \left(1 + \frac{d}{R}\right)^{-1}.$$

Expanding (see Note 3, page 192),

$$\cos D = 1 - \frac{D^2}{2!} + \frac{D^4}{4!} - \cdots \text{ and} \left(1 + \frac{d}{R}\right)^{-1}$$
$$= 1 - \frac{d}{R} + \frac{d^2}{R^2} - \cdots$$

and since both D and $\frac{d}{R}$ are small we may retain only the first

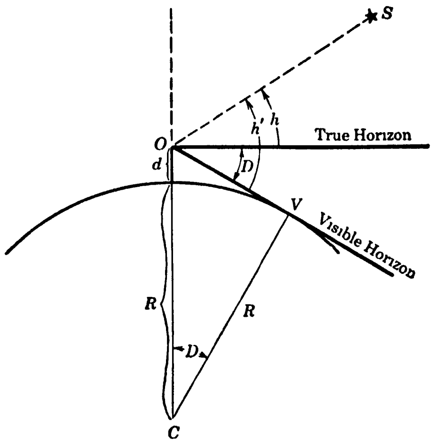

FIG. 29.—Dip of the horizon.

two terms of each expansion. Therefore

$$1 - \frac{D^2}{2} = 1 - \frac{d}{R}$$

or

$$D \text{ (in radians)} = \sqrt{\frac{2d}{R}}.$$

Since 1 radian = 3438′ and R = 20,900,000 ft., we have

$$D' = 1.064\sqrt{d}.$$

This formula, however, does not take into account the fact that due to refraction the visible horizon seems *higher*, and thus the dip is *less* than the geometrical drawing shows. An approximate formula for the dip with allowance for the effect of refraction is

$$D' = \sqrt{d}. \tag{34}$$

That is, the *dip in minutes of arc is equal to the square root of the observer's elevation in feet.*

50. Parallax (geocentric) is the angle *at the heavenly body* formed by two lines, one drawn to the center of the earth and the other to the place of observation. In other words, it is the difference in direction of the heavenly body as seen from the center of the earth and from the place of observation. When the heavenly body is at the horizon this angle is maximum (considering the earth a sphere) and is called the *horizontal parallax* (Fig. 30).

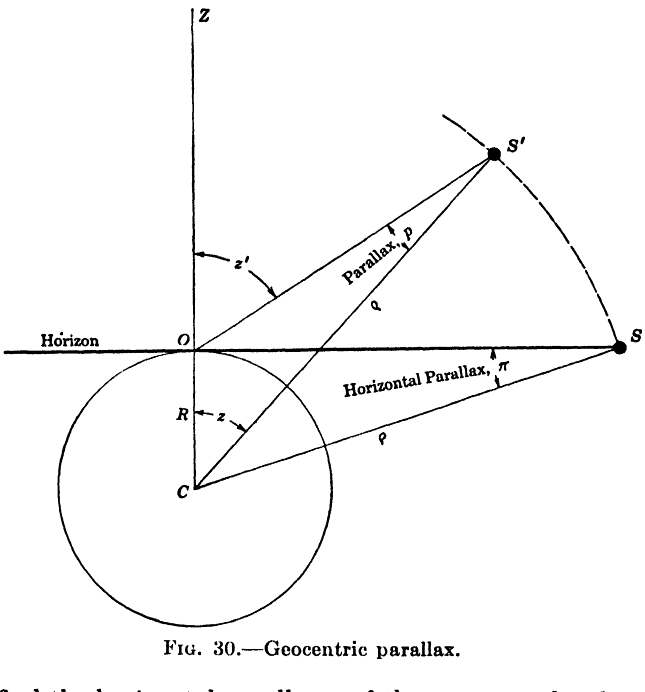

FIG. 30.—Geocentric parallax.

To find the horizontal parallax π of the sun, consider the earth as a sphere and let R be the earth's radius and ρ the sun's distance from the center of the earth; then from Fig. 30

$$\sin \pi = \frac{R}{\rho}. \tag{35}$$

To find the parallax p of the sun at *any* observed *zenith distance* z', let z represent the corresponding geocentric zenith distance and from triangle $OS'C$ we have

$$\frac{\sin p}{R} = \frac{\sin (180^\circ - z')}{\rho}$$

or

$$\sin p = \frac{R}{\rho} \sin z' \tag{36}$$

and from Eq. (35)

$$\sin p = \sin \pi \cdot \sin z'. \tag{37}$$

The horizontal parallax of the sun is given in the American Ephemeris for every day in the year (see page 44 of this book); it is never greater than 8.″95 and the average value is 8.″8. Since the value of p is less than this, we may write in the place of Eq. (37)

$$p = 8.''8 \sin z'. \tag{38}$$

We see from Fig. 30 that

$$z' = z + p.$$

Hence,

$$z = z' - 8.''8 \sin z'. \tag{39}$$

That is, the *parallax correction should be subtracted from the observed zenith distance.*

Similarly the parallax correction for the observed zenith distance z' of the moon may be obtained. From Eq. (37) we have

$$\sin p_m = \sin \pi_m \cdot \sin z'.$$

Hence

$$p_m = \pi_m \cdot \sin z' \tag{40}$$

when π_m is the horizontal parallax of the moon, which is tabulated in the American Ephemeris.

The mean horizontal parallax ($\bar{\pi}_m$) of the moon may be obtained from its average distance from the center of the earth and Eq. (35). That is

$$\sin \bar{\pi}_m = \frac{3,959}{239,000} \text{ and } \bar{\pi}_m = 56' \, 57''.$$

51. Semidiameter.—The angle made at the center of the earth by the sun's semidiameter is known as the *angular (geocentric) semidiameter of the sun.* If ρ represents the distance from the center of the earth to the center of the sun and r the mean radius of the sun, then we have from Fig. 31 for the angular semidiameter of the sun S,

$$\sin S = \frac{r}{\rho}$$

or since S is small

$$S \text{ (in radians)} = \frac{r}{\rho}.$$

The Ephemeris gives the value of S for every day in the year; it also gives the angular semidiameter of the moon for 0^h and 12^h G.C.T. of each day.

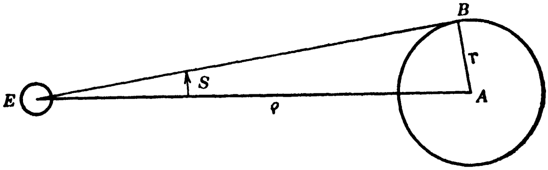

FIG. 31. – Relation between angular and linear semidiameter of the sun or moon and their distance from the earth.

a. Correction To Be Applied to the Observed Altitude or Zenith Distance.—Since in making observations of altitude on the sun or moon it is difficult to sight with accuracy at the center, the *limb* or edge is observed. This limb is well defined and the setting can be made with great exactness. The position of the center is obtained by correcting the observation for angular semidiameter. This is somewhat different for different altitudes, on account of the fact that the object is nearer to the observer than it is to the center of the earth (Fig. 30). In the case of the sun, this difference is too small to be considered; however, in the case of the moon the difference may be as large as 15″ and must be considered.

Let S be the semidiameter of the sun as obtained from the Ephemeris, and z' the observed zenith distance of its upper or lower limb (corrected for refraction); then the corresponding zenith distance of the center of the sun is

$$z = z' \pm S \tag{41}$$

When the *upper limb* is observed the *plus sign* must be used; and when the *lower*, the *minus sign*.

b. Correction to Be Applied to Angles Measured in the Horizon.—To obtain the azimuth of a line from observations on the sun (Art. 99) the *horizontal angle* from the line to the eastern or western limb of the sun is measured. This angle is usually recorded clockwise from the line and is denoted here by A'. Figure 32 shows the vertical circle ZD tangent to the limb of the sun at B, and the vertical circle ZE through its center C. The

angle A' must be corrected by the angle s to reduce the observation to the center. In the right spherical triangle ZBC the angle at Z is equal to s, the semidiameter S is obtained from the Ephemeris, and the side opposite to the right angle is the zenith distance z of C. From the law of sines,

$$\frac{\sin s}{\sin S} = \frac{\sin 90°}{\sin z}.$$

Since both s and S are small angles, we may write,

$$s = \frac{S}{\sin z}. \qquad (42)$$

The correction, s, is to be added to the horizontal angle, A', when the eastern limb of the sun is observed, and subtracted when the western limb is observed. From Eq. (42) we see that z must

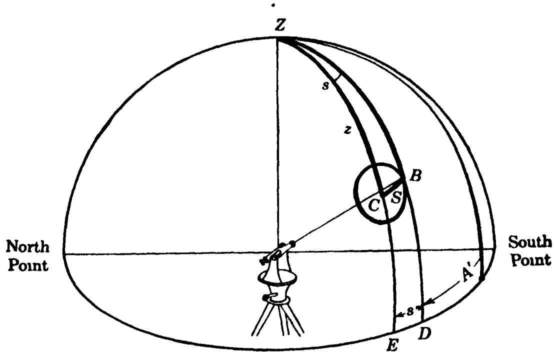

FIG. 32.—Horizontal angle correction for sun's angular semidiameter.

not be very small, hence the sun must not be observed near the zenith.

52. Personal Equation.—The error due to the inability of an observer to determine the exact instant at which a star crosses a certain wire in the field of his telescope is more or less systematic and is known as *personal equation.* It is a troublesome correction as it differs not only for different observers but also for varying physical conditions of the same observer and varying brightness of stars. It is usually less than one-tenth of a second of time and hence too small to be included in ordinary observations with the engineer's transit, the sextant, or theodolite. It is applied, if known, when accurate determination of time is made with the astronomical transit (Chap. XII).

53. Sequence of Corrections to Observed Altitude.—The corrections considered in this chapter mainly refer to observations of altitude made with the engineer's transit, the sextant, or theodolite. They should be applied to the observed altitude in the following order: (*a*) dip of the horizon, (*b*) refraction, (*c*) semidiameter, and (*d*) parallax. However, inasmuch as the instruments used do not attain a high degree of precision, they may be applied in any order. The altitude or zenith distance thus corrected is usually known as the *true* altitude or zenith distance. In the case of land observations on a star the only correction to be applied is that for refraction.

If the mean of a large number of observations is used for the altitude, the above corrections may be computed to the nearest tenth of a second; otherwise they should be carried to the nearest second.

Exercises

1. Compute the corrected zenith distance of a star, given the observed altitude equal to 21° 0′ 30″.

2. The height of a ship's bridge above sea level is 50 ft. Find the true altitude of a star when its altitude as observed from the bridge is 35° 12′.

3. What is the distance in miles of the horizon to an observer at an altitude of 400 ft?

4. What is the greatest horizontal parallax of Mars? Use, for the required parts, mean values given on page 2.

5. Find the angular semidiameter of the moon using the mean distance and mean diameter of the moon given on page 2.

6. What is the true zenith distance for the center of the sun on July 1, 1929, when the altitude of its lower limb as observed with the engineer's transit (free from instrumental errors) is 32° 17′ 0″?

7. The altitude of the sun's lower limb when observed from a ship's bridge (45 ft. above sea level) on June 1, 1929, was found to be 25° 26′ 10″. Find the true altitude of the center assuming that there were no instrumental errors.

8. The meridian altitude (south of the zenith) of the moon's lower limb is 34° 50′. The declination of the center of the moon is +5° 12′ 10″. Find the latitude of the observer. (Use the mean radius of the earth and the moon and their mean distance given on page 2.)

9. Find the latitude of the observer and the declination of a north circumpolar star, given:

Observed altitude at upper transit = 55° 42′ 10″.
Observed altitude at lower transit = 21° 11′ 45″.

10. Find the corrected zenith distance and azimuth of the center of the sun, at the instant the observed altitude is 40° 47′ 30″ and the azimuth of the eastern limb is 302° 12′ 15″, on July 1, 1929.

CHAPTER VII

INSTRUMENTS

The engineer's transit, the sextant, and the theodolite are the principal instruments used in the work of practical astronomy outlined in this book. The complete theory of these instruments together with their adjustments is outside the scope of this book; however, a general description of them and of their ordinary adjustments will be given. A discussion will also be given of the attachments used with these instruments to facilitate astronomical observations. The solar attachment used with the engineer's transit will be described in Chap. XI.

54. The Engineer's Transit.—It is assumed that the student is familiar with the appearance and ordinary use of this instrument. It is composed essentially of a *horizontal plate* and reading circle carrying one or two levels, two vertical standards resting on the plate to support the *horizontal axis* on which a *telescope* and a *vertical circle* are mounted. A level is usually attached parallel to the telescope tube. The tangent at the center of the inner surface of the level tube is called the *axis of the level*. The horizontal plate is accurately made perpendicular to the axis of the spindle which carries the weight of the entire instrument; this axis is the *vertical axis* of the instrument.

In the focal plane of the objective of the telescope are placed two perpendicular *cross-wires*. The line joining the geometrical center of the objective and the point of intersection of the cross-wires is known as the *line of sight*. If the engineer's transit is to be in perfect adjustment the following conditions must be obtained:

a. The axes of the plate levels must be perpendicular to the vertical axis of the instrument.

b. The vertical cross-wire must be in a plane perpendicular to the horizontal axis.

c. The line of sight must be perpendicular to the horizontal axis.

d. The vertical and horizontal axes must be perpendicular.

e. The axis of the level attached to the telescope, and the line of sight, must be parallel.

f. When the line of sight is horizontal the vertical circle must read zero.

On account of mechanical imperfections it is highly improbable that any of these ideal conditions will be fulfilled. They are regarded as errors and made as small as possible by careful adjustment. Their elimination is sought by suitable methods of observation and reduction. For example, to eliminate the error introduced to a measurement of altitude by the fact that the vertical circle does not read zero when the line of sight is horizontal, two observations are made, one with the telescope direct and the other with the telescope reversed, and their mean taken. There is of course 180° difference between the two corresponding circle readings due to the reversal itself.

55. Adjustments of the Engineer's Transit. *a. To Make the Axes of the Plate Levels Perpendicular to the Vertical Axis.*—Set up the instrument, bring each of the levels parallel to a line joining two of the opposite leveling screws, and bring both bubbles to the center by means of the corresponding leveling screws. *Rotate* 180°; correct half the error in each level by means of the leveling screws and the other half by means of the adjusting screw at the end of the tube of each level. Repeat if necessary.

b. To Bring the Vertical Cross-wire into a Plane Perpendicular to the Horizontal Axis.—First obtain a sharp focus of the cross-wires, by pointing the telescope to the sky. The wires are properly focused when there is no relative movement between them and a distant object as the eye is moved across the eyepiece. Level the instrument and by means of the vertical wire bisect some small distant object at the upper edge of the field of view, move the telescope in altitude, and see whether or not the object remains bisected. If not, adjust by means of the four screws that hold the ring on which the cross-wires are fastened.

c. To Make the Line of Sight Perpendicular to the Horizontal Axis of the Telescope.—Level instrument and bisect with the vertical wire an object a few hundred feet away. Clamp the plates and reverse telescope on its horizontal axis, and bisect another object at about the same distance from the instrument. The two objects are chosen approximately level with the instrument. Turn the instrument in azimuth and sight at the first object. Again clamp the plates and reverse the telescope on its axis. If the cross-wire bisects the second point, no adjustment is

necessary; if not, move the cross-wires laterally one-fourth of the amount of the deviation by means of the screws which hold the ring of the cross-wires. Repeat in the same order, also repeat adjustment *b*.

d. To Make the Vertical and Horizontal Axes Perpendicular.—Suspend a plumb line 20 to 25 ft. long, about 15 ft. from the instrument. Immerse the weight in a pail of water to avoid excessive swing of the line. Level the plate *carefully* and bisect the upper part of the line. If the plumb line remains bisected throughout while the telescope is moved in altitude, no adjustment is necessary. Otherwise, adjust by raising or lowering the adjustable end of the horizontal axis.

e. To Make the Axis of the Level Attached to the Telescope Parallel to the Line of Sight.—Level the instrument half way between two points about 300 ft. apart and nearly on the same level with it. Drive a stake at one of these points, bring the level bubble to the center of the tube and obtain the reading on a rod set on the stake. Direct the telescope toward the other point, again bring the bubble to the center of the tube and drive a stake near the second point until the rod over this stake gives the same reading as it gave over the first stake. Now the tops of the two stakes are at the same level. Set the instrument outside of the line joining the two and as near to the first stake as it can be, and still focus clearly on it. Level, and read the rod set on the first stake when bubble of level of telescope is centered. Likewise read rod on second stake. The difference of these two readings must be made zero. Raise or lower the telescope by means of its tangent screw so that the reading at the distant stake is the same as that at the nearer stake. With the telescope still clamped, center the level bubble by raising or lowering the adjustable end of its tube.

f. When the Line of Sight Is Horizontal the Vertical Circle Must Read Zero.—Having made adjustment *e*, bring the bubble of the level attached to the telescope to the center of its tube and adjust the vernier of the vertical circle to read zero. If it is impossible to make this adjustment, the reading of the circle at this position, known as the *index error*, is applied as a correction to all vertical-angle readings made.

56. Reflector.—When the engineer's transit is used for night observations, various means are used to illuminate the cross-wires. Perhaps the simplest method is to throw light from a flashlight

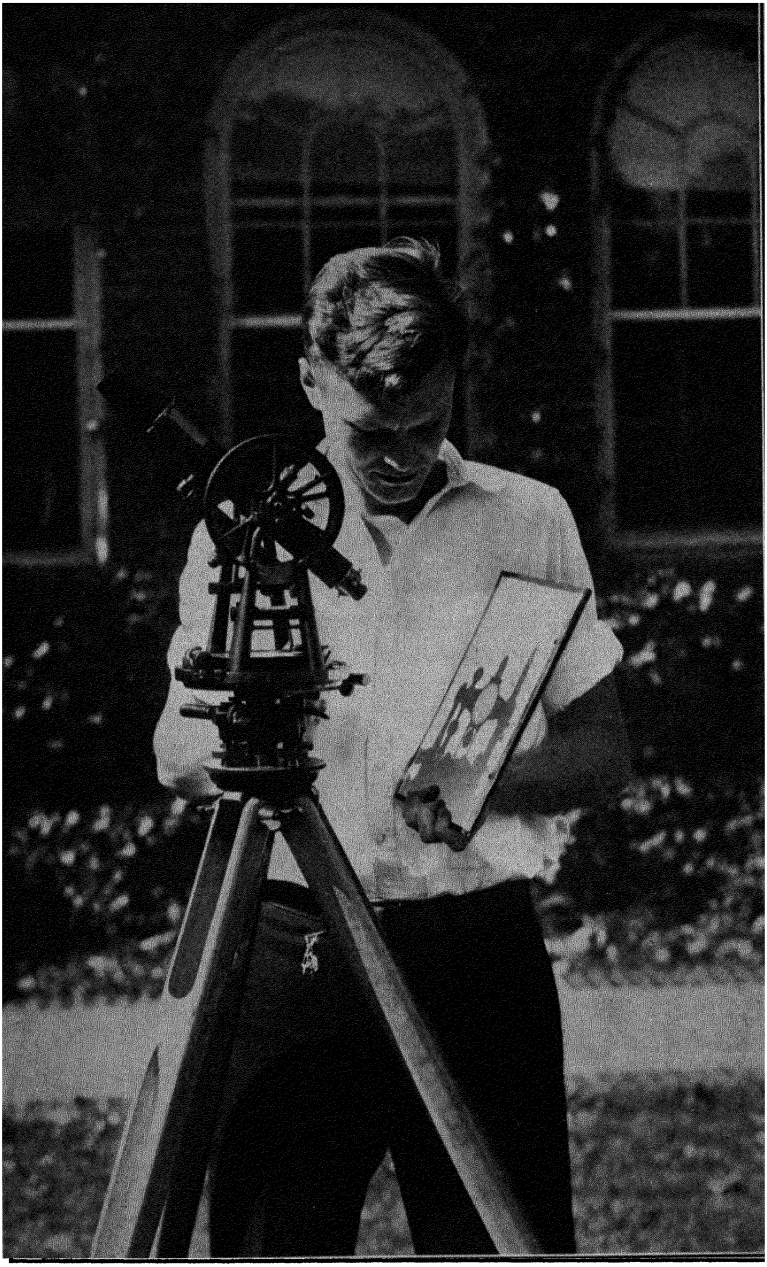

FIG. 33.—Observing the sun by projecting its image and the image of the cross-wires of the transit on a piece of paper.

down the telescope tube through the objective, being careful not to obstruct the line of sight. The cross-wires appear dark against the illuminated field.

Most transits are fitted with a special *reflector* which is mounted on a ring that fits around the objective and is inclined about 45° to it; an opening in the center of the reflector allows the use of the telescope. The light for illuminating the cross-wires is so held near the inner surface of the reflector that it does not shine into the face of the observer.

57. Prismatic Eyepiece.—Such an eyepiece is used with the engineer's transit in order that great vertical angles may be observed. It consists of a prism which may be attached to the eyepiece and which reflects light at right angles; it makes possible the measuring of altitudes as great as 70°. The prismatic eyepiece inverts the image in the up-and-down direction but does not invert it right and left, so that, when the sun or the moon is observed, care must be exercised not to confuse the upper and lower limb.

When the telescope is directed toward the sun, a dense colored glass attached to the prismatic eyepiece is moved over its opening to protect the eye from the sunlight. *Never attempt to observe the sun without such protection.* An equally satisfactory result may be obtained without the colored glass, by projecting upon a piece of paper (Fig. 33) the images of sun and cross-wires as follows:

Bring the sun into the field of view. Pull out the eyepiece and move the paper toward or away from the eyepiece until the image of the wires is sharp. Then, by means of the focusing screw, secure a sharp edge of the sun's image.

58. The sextant is an instrument for measuring angles, especially useful because it requires no fixed support, is light, and is easy to handle. It is well adapted for observations at sea and for reconnaissance and hydrographic survey.

The sextant consists essentially of a graduated arc ABC (Figs. 34 and 35) of radius about six inches, with center at O. An *index arm* OB is pivoted at O and carries the *index mirror* M perpendicular to the plane of the arc. In the line of sight of the telescope T is the *horizon-glass* H, fixed at right angles to the plane of the arc and *parallel* to the index mirror when the index arm is set to read zero. The lower half of the horizon-glass H (*i.e.*, the half nearer to the plane of the arc) is silvered, while the

upper half is left clear. One set of colored shades is mounted near the index mirror and can easily be brought in front of it to reduce the excessive brightness when observing the sun; a similar set

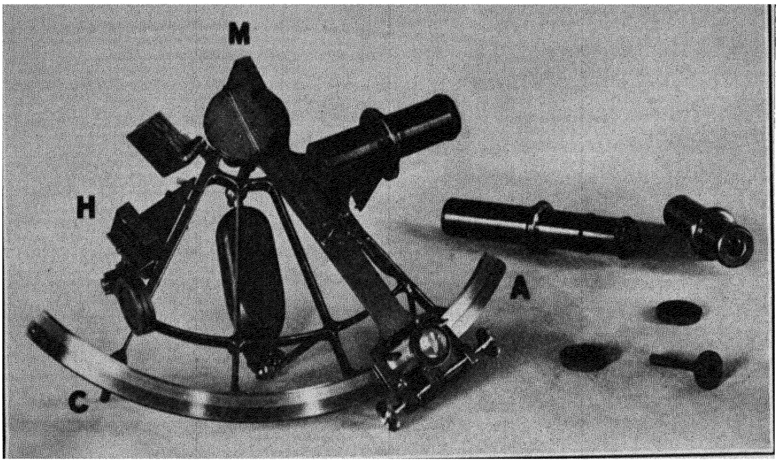

FIG. 34.—The sextant. (*Courtesy of The Warner and Swasey Co.*)

is placed near the horizon-glass. The arc is graduated to 10′ and the vernier reads to 10″.

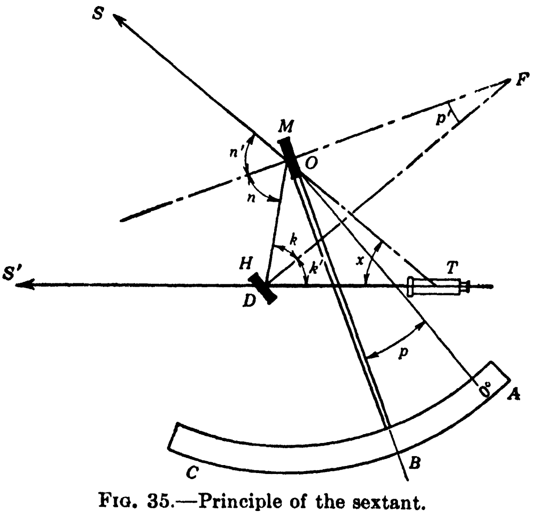

FIG. 35.—Principle of the sextant.

59. The Principle of the Sextant.—Let us suppose the instrument in perfect adjustment. Then, to illustrate the principle of

the sextant let us measure the angle STS' (Fig. 35) between two stars or two distant terrestrial objects S and S'. Point the telescope at the star S'. The pencil of light $S'T$ comes through the clear part of the horizon-glass H and forms an image of the star S' at the focal plane of the telescope. Rotate the sextant about the axis of the telescope so that the star S comes into the plane of the instrument. Slowly swing the index arm until the pencil of light from S following the path $SODT$ is brought into the field of the telescope, forming an image in the focal plane. Clamp index arm and bring the images of the two stars into coincidence by means of the slow-motion screw. The circle is so graduated that the reading obtained gives the angle STS'. *It is evident that the two images will not coincide unless S and S' are in the plane of the instrument.* This fact permits accurate measurements to be made with the sextant without requiring a firmly fixed support.

If OF and FD represent the normals to the surfaces of the mirrors M and H, respectively, we have from optics

$$n = n' \text{ and } k = k',$$

and from the triangles FDO and ODT,

$$p' + k = n \text{ and } 2k + x = 2n$$

where x is the angle STS', and p' the angle between the normals.

Eliminating n from these equations we have

$$p' = \frac{x}{2}.$$

If p represents the angle of rotation of the index mirror, then $p = p'$, since they have their sides respectively perpendicular. Hence,

$$p = \frac{x}{2},$$

from which we see that to be able to read directly on the arc of the sextant the angle between the two objects whose images coincide (*i.e.*, the angle x) the graduations of *half degrees* are *numbered as whole degrees* (Fig. 36).

60. Adjustments.—The following adjustments should be made in the order given:

a. To Make the Index Mirror Perpendicular to the Plane of the Sextant.—Remove the telescope and place the instrument on a table with the arc away from you. Set index at about the

middle of the arc and look into the index mirror in such a way as to see, at the same time, part of the arc by reflection and part direct. If the two images form a continuous arc, no adjustment is necessary; if not, tip the mirror by whatever means has been provided on that particular instrument.

b. To Make the Line of Sight of the Telescope Parallel to the Plane of the Sextant.—Place the instrument on a table about twenty feet from a wall and *sight along the arc.* Mark on the wall a line in the plane of the arc. Measure the distance from the center of the telescope to the plane of the arc and mark a second line on the wall as much above the first as the distance just measured. Usually the telescope of the sextant is provided with four cross-wires to mark the approximate center of the field. Rotate the eyepiece until two of the parallel wires are horizontal. Now see if the image of the second mark bisects the space between the parallel wires. If so, the telescope is adjusted; if not, adjust by means of the screws in the ring which carries the telescope.

c. To Make the Horizon-glass Perpendicular to the Plane of the Sextant.—Point the telescope to a well-defined object: a star, the sun, or a distant terrestrial mark. Move the index arm slowly back and forth past the zero mark. If the reflected image of the object does not coincide with the direct in passing it, the horizon-glass is not perpendicular to the plane of the sextant. Adjust by means of the screw provided for that purpose at the back of the horizon-glass.

d. To Make the Two Images Equally Distinct.—If the direct image of the object is brighter than the index image, the telescope receives more light through the plain glass and, therefore, the distance between the telescope and the frame of the instrument should be decreased. The adjustment is made by means of a screw in the reverse side of the frame.

e. To Make the Vernier Read Zero When the Index Mirror and Horizon-glass Are Parallel.—The error arising from this is usually called the *index error.* Set index at zero and observe a very distant, well-defined object. If the two images coincide, no adjustment is necessary. If not, turn the horizon-glass about the axis perpendicular to the frame by means of the proper screw, until the two images coincide.

61. The Index Correction.—Since the index error cannot be depended on to remain zero or even constant from day to day,

it is usually *determined before each series of observations.* The amount to be *added* to the measured angle is called the *index correction* (i).

The index of the vernier is said to be *on the arc* when it is to the *left* of the zero mark of the graduated arc, and *off the arc* when it is to the *right.* To determine the index error, make the direct and reflected images of a star or a well-defined distant object coincident and observe the reading R of the arc. If the index is *on the arc* we shall have for the index correction:

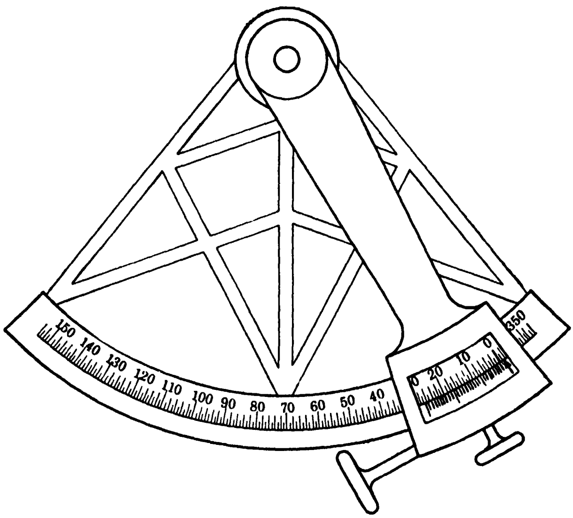

FIG. 36.—The vernier index is to the right of the zero mark and hence it is said to be "off the arc." The approximate reading of the angle shown is 359° 15′.

$$i = 0° - R, \text{ that is, } i \text{ is negative} \tag{43}$$

When the index is *off the arc:*

$$i = 360° - R, \text{ that is, } i \text{ is positive} \tag{44}$$

To avoid confusion in reading the vernier when the index is off the arc, the angle is not read negative but as positive and nearly 360° (Fig. 36).

Example: Find the index correction (i) from the following six readings on a star.

°	′	″	°	′	″
359	57	00	359	57	10
	56	40		56	50
	57	10		57	40

The mean of the six readings is 359° 57′ 5″, and hence,

$$i = 360° - 359° 57' 5'' = +2' 55''.$$

For solar observations point the telescope to the sun and make the two images externally tangent. Never observe the sun without protecting your eye with the dense shades of the sextant. First, put the direct image *below* the reflected image, in which case the vernier index will be on the arc (Fig. 37). It is assumed

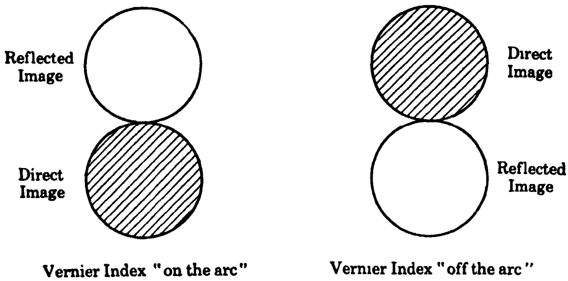

FIG. 37.—Position of the two images of the sun for determining the index error of sextant, using the inverting telescope.

here that the inverting telescope is used. The angular diameter of the sun (D) will equal the vernier reading R_1 plus i, *i.e.*,

$$D = R_1 + i.$$

Second, put the direct image *above* the reflected image, in which case the vernier index will be off the arc. If R_2 is the vernier reading, $360° - R_2$ expresses the angular diameter of the sun, provided the index error is zero. However, in this case the index error affects the result in the opposite direction and, hence,

$$D = (360° - R_2) - i.$$

Eliminating D and solving for i, we have

$$i = 180° - \tfrac{1}{2}(R_1 + R_2) \qquad (45)$$

Example: The following readings were taken on the sun for the determination of index correction:

	On the Arc				Off the Arc		
	°	′	″		°	′	″
	0	36	10		359	33	20
			20				40
			30				30
Mean for R_1 =	0	36	20	Mean for R_2 =	359	33	30

$$i = 180° - 180° 4' 55'' = -4' 55''.$$

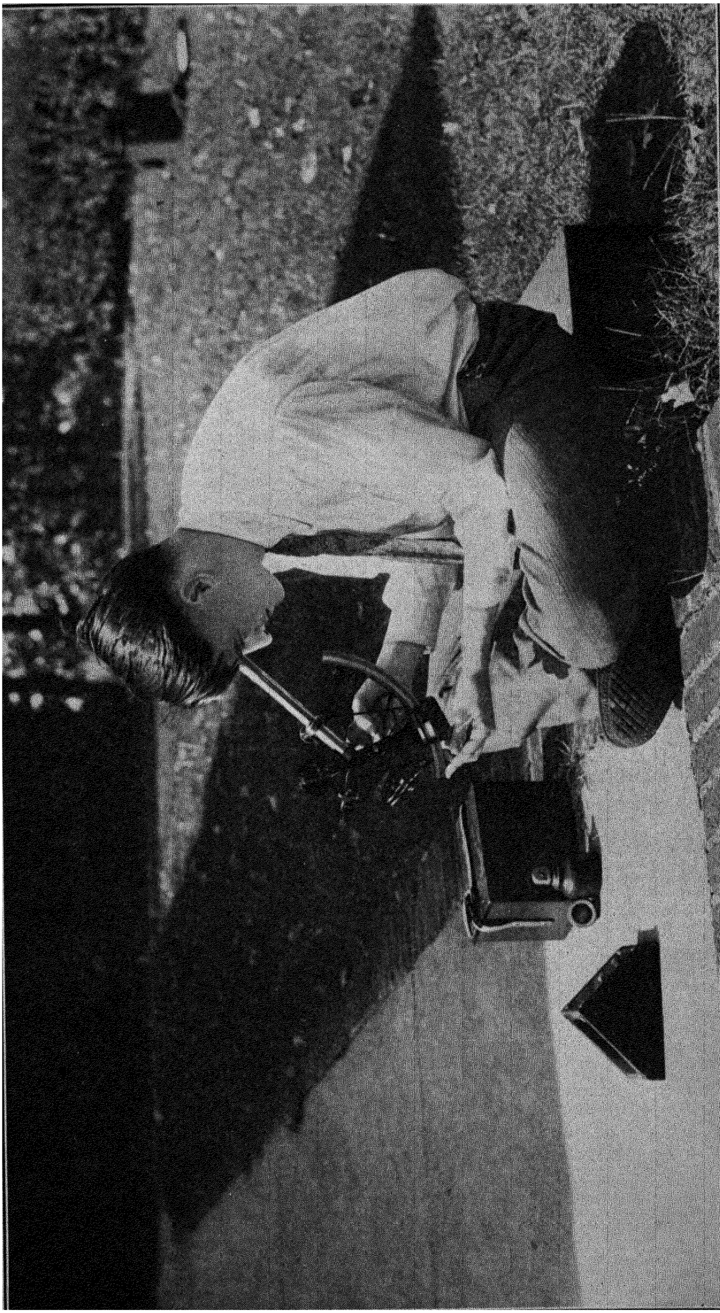

FIG. 38.—Observing the sun with the artificial horizon. The telescope is pointed at the mercury horizon. The index finger of the left hand supports the arc and the thumb and middle fingers operate the clamp and slow motion.

62. Measuring Altitude with the Sextant.—When the sextant is used at sea, the altitude of a star is obtained as follows: holding the instrument in the right hand, point the telescope at the horizon directly below the star and move the index arm so that the reflected image of the star is brought to the horizon in the middle of the field. To bring the instrument into a vertical

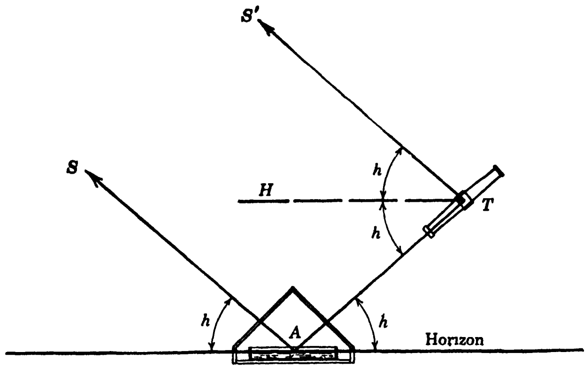

FIG. 39.—Double altitude. The angle that the ray of incidence SA makes with the mercury horizon is the altitude of the star and is equal to the angle that the reflected ray AT makes with the horizon. If TH is drawn parallel to the horizon the angle HTS' is also equal to the altitude of the star, hence the angle ATS' which is measured by the sextant is *twice the altitude.*

plane, rotate it slightly about the axis of the telescope by a small movement of the wrist. This will cause the star image to describe an arc. By means of the slow-motion screw make the arc tangent to the horizon. When the altitude is changing rapidly, as it is in case of stars away from the meridian, proceed thus: for a star east of the meridian, with the slow-motion screw slightly *increase* the altitude and wait until the arc described by the image of the star is tangent to the horizon; for stars west of meridian, *diminish* the altitude and wait for the tangency as before.

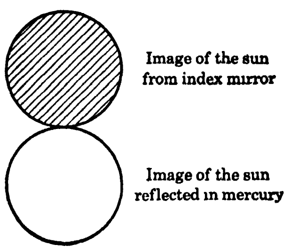

FIG. 40.—When the altitude of the *upper* limb of the sun is observed with the inverting telescope, the image reflected in the mercury appears *below* the image from index mirror.

When observing the sun, *the color shades are placed in front of the index mirror and horizon-glass to diminish the brightness of the field.* The arc described by the lower limb of the sun is made tangent to the horizon.

When the altitude of a heavenly body is measured on land, the *artificial horizon* is used. This is a rectangular shallow basin of mercury, protected from wind by a sloping roof of glass. The observer so places himself (Fig. 38) that he can see the image of the body whose altitude is to be measured reflected in the mercury, and points the telescope toward the artificial horizon

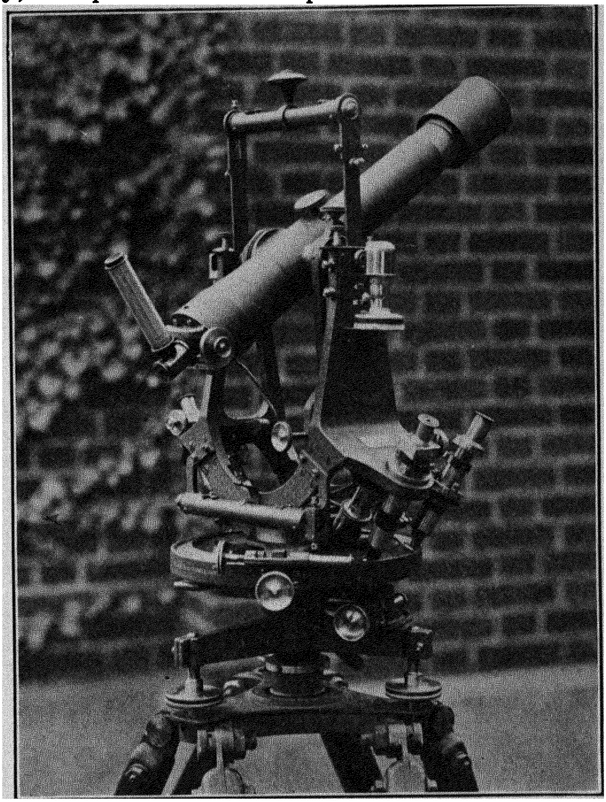

FIG. 41.—The theodolite. The diameter of the horizontal circle of this theodolite is 8 inches, the horizontal circle is read by two reading microscopes to seconds, the third reading microscope gives the degrees and the nearest five minutes.

so that for "direct image" he uses a second image of the heavenly body, in the place of the horizon. By moving the index arm, the two images are made to coincide. The reading of the circle gives *twice the altitude;* this is clear from Fig. 39. The index correction must be applied to the *double angle.*

When the sun's altitude is measured the two images are made tangent externally or allowed to move into external tangency.

This gives the double altitude of the upper or lower limb. Suppose that the inverting telescope is used and that the image reflected from the mercury is made red by means of the red color shade, and the other image blue. Then if the red image of the sun is below (Fig. 40), the *upper* limb of the sun has been observed, and if the red image is above, the *lower* limb.

63. The Theodolite.—This name is usually given to an instrument designed to measure horizontal and vertical angles much more precisely than the engineer's transit, although the name is reserved more correctly for instruments in which the telescope cannot be reversed without being lifted from its supports. A theodolite (Fig. 41) is mainly used for the accurate measurement of horizontal angles. When it is provided with a large vertical circle, it is called an *altazimuth instrument* and is especially adapted for astronomical observations for latitude and azimuth.

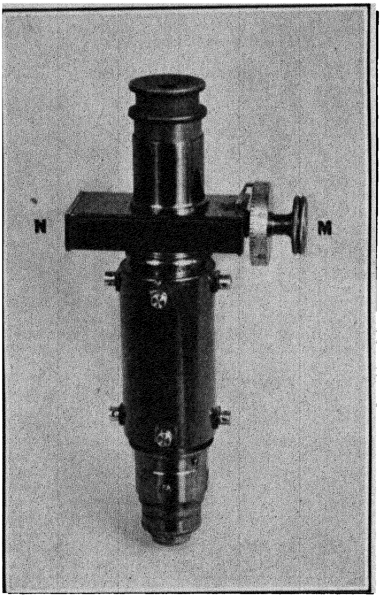

FIG. 42.—The reading microscope. (*Courtesy of The Warner and Swasey Co.*)

In general the theodolite is larger than the engineer's transit and has three leveling screws and a horizontal circle of 8 to 12 in. diameter graduated with 5′ or 10′ divisions. The horizontal angles are read either by vernier to 5″ or 10″ or by micrometer microscopes (Art. 64) to a single second. The plate level or levels are more accurate than those of a transit, and a striding level (Art. 66) is provided for the horizontal axis of the telescope. The standards are short and the telescope much larger than on a transit. The adjustments are much the same as those of the engineer's transit.

To level instruments with three leveling screws, set the level parallel to two of them and bring the bubble to the center by turning these screws equally and in opposite directions; turn the level perpendicular to the same pair of leveling screws and center the bubble by the third screw.

64. The Reading Microscope.—When it is required to read the horizontal or vertical circle of the theodolite closer than 5″, the vernier is replaced by the reading microscope (Fig. 42), which reads to single seconds. It consists essentially of a microscope which forms an image of the graduations at the focal plane *MN* of the objective. In this plane a pair of parallel fine wires (spider lines) *AB* (Fig. 43) are placed, and made to move by means of a micrometer head *M*. The graduations and wires are observed with a magnifying eyepiece. The micrometer screw has a very fine thread and the head is graduated into 60 parts. The circle of the theodolite is usually graduated into

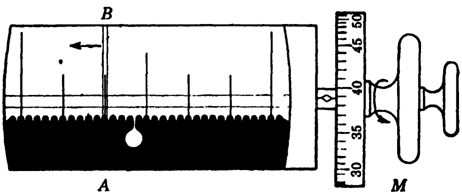

FIG. 43.

5′ spaces and when the microscope is properly adjusted five turns of the screw will move the wires over one space. Hence, one complete revolution of the screw carries the parallel wires 1′ and one division of the screw head corresponds to 1″. To facilitate the counting of the whole turns there is in the field of view a sawtoothed index on which the distance between adjacent teeth corresponds to one turn of the screw. The center of the field is marked by a circle on this index. When the micrometer head is turned in the direction of *increasing* reading (positive direction), the micrometer wires move in the direction of *increasing* reading of the graduations of the limb.

To read an angle, turn the micrometer head in the *positive direction* from the center of the field to the nearest graduation of the limb. The number of complete turns is obtained by counting the depressions of the sawteeth which will give the minutes; and the reading of the micrometer head, the number of seconds. Figure 43 shows the reading 3′ 38″.7, the 3′ being given by the sawtoothed index, the 38″.7 by the micrometer head. To avoid lost motion in the screw, the exact placing of the wires over the graduation is made in the positive direction. The degrees and the nearest 5′ are read directly by means of the *index micro-*

scope which is of lower power and larger field and is mounted on another part of the limb (Fig. 41).

65. The Error of Runs of the Reading Microscope.—As in the previous paragraph, suppose we consider an instrument with horizontal circle divided to 5′ of arc. If the microscope is properly focused and adjusted, five complete turns of its screw will

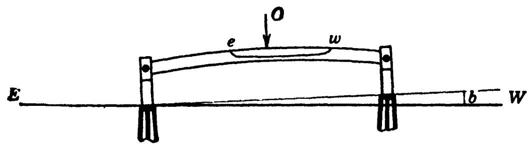

FIG. 44.

carry it from one division of the limb to the next. It is evident that this can be realized only approximately. The discrepancy is known as the *error of runs* and is determined from time to time by measuring 5′ spaces in different parts of the circle. To illustrate, let the average reading of 20 divisions be 5 revolutions

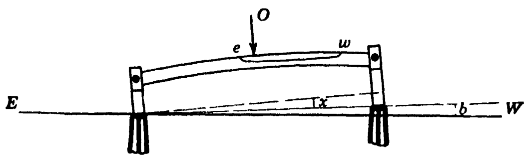

FIG. 45.

and 1.5 divisions of head. This will give for error of runs per minute $1''.5 \div 5 = 0''3$. Suppose it is required to correct the measured angle of 95° 44′ 30″.2. The correction is to be applied to 4′ 30″2 and it is equal to $-1''.4$. Hence the angle corrected for runs is 95° 44′ 28″.8.

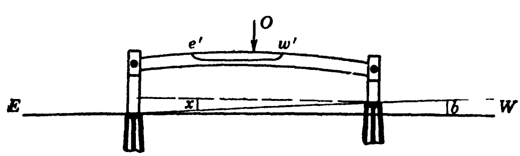

FIG. 46.

66. The Spirit Level.—The inclination of the rotation axis (horizontal axis) of a theodolite or astronomical transit (Chap. XII) is determined with a spirit level. It consists of a closed glass tube, nearly filled with alcohol or ether. The upper inner side of the tube is accurately ground so that the longitudinal section is an arc of a circle of large radius. The bubble of

air formed in the tube has the tendency to occupy the highest position of the arc so that a small change in the inclination of the tube will cause a motion of the bubble. A graduated scale on the surface of the glass or on the frame holding the tube indicates the position of the bubble. Some levels have the zero at the center and are numbered both ways from it. The more modern levels have the zero at one end. Figure 44 shows an ideally adjusted level graduated with the zero at the center, resting on the supports of the axis of rotation of an instrument. For convenience in the following description, this axis is placed in an east-and-west direction.

Let

w = reading of west end of bubble.

e = reading of east end of bubble.

d = the given angular value of one division of the scale.

b = the vertical angle between the horizon EW and the line joining the ends of the supports.

The center of the bubble will be $\frac{1}{2}(w - e)$ divisions from the zero. This expression will be positive when the west support is higher, and negative when lower. Hence,

$$b = \tfrac{1}{2}(w - e)d.$$

Usually the level is not ideally adjusted: Fig. 45 shows this case.

Let x be the error of adjustment expressed as an angle. Then,

$$b + x = \tfrac{1}{2}(w - e) \cdot d. \tag{46}$$

To eliminate x, the level is reversed as shown in Fig. 46. If e' and w' denote the reading of the east and west ends of the bubble, respectively, for the level reversed, we have:

$$b - x = \tfrac{1}{2}(w' - e') \cdot d. \tag{47}$$

Adding the last two equations, we have:

$$b = \tfrac{1}{4}[(w + w') - (e + e')] \cdot d. \tag{48}$$

It is therefore evident that to *obtain the inclination of the axis of rotation of an instrument, we must read the level direct and reversed.*

If the zero of the graduations of level is at one end of the tube, we have:

$$b = \tfrac{1}{4}[(w + e) - (w' + e')] \cdot \mathrm{d}. \tag{49}$$

In this case, we consider level direct when zero of level is east.

Example: The following readings of the level were made at the Warner and Swasey observatory for determining the inclination of the axis of the 3-in. astronomical transit ($d = 0''\!.85$):

Level Direct		Level Reversed	
w	e	w'	e'
82.5	71 4	92 5	103.6

From Eq. (49) we have:

$$b = \tfrac{1}{4}(153.9 - 196.1) \cdot 08''\!.5 = -8''\!.97.$$

The negative sign indicates that the west support of the instrument is *lower* than the east. The length of the bubble is the difference between the east and west readings. In this case it is 11.1 divisions for both the direct and reversed position. It is well to check this when level readings are taken.

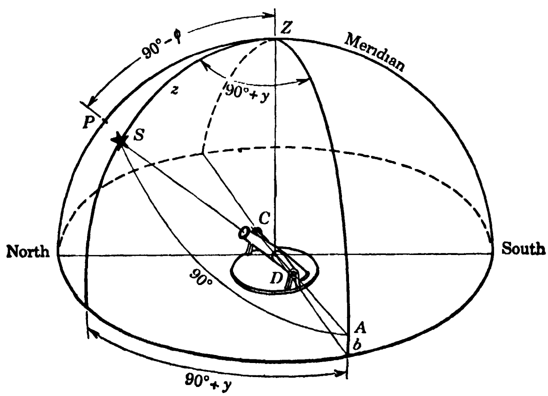

FIG. 47.

***67. Correction for Level.**—When the axis of rotation of a theodolite is not horizontal the line of sight will not describe a vertical circle and therefore the horizontal-angle reading will require a small correction. Figure 47 shows the axis of rotation CD when produced piercing the celestial sphere at A and the telescope pointing to the star S. The angle that CD makes with the horizon is b, while AS is an arc of a great circle and equal to 90°, as it measures the angle between the horizontal axis of the instrument and the line of sight. If y is the required correction, we have:

$$\text{Angle } AZS = 90° + y,\ AS = 90°,\ ZS = z,\ ZA = 90° - b.$$

From the law of cosines of the triangle AZS, we have:

$$\cos 90^\circ = \cos (90^\circ - b) \cos z + \sin (90^\circ - b) \sin z \cos (90^\circ + y),$$

or

$$\sin y \sin z \cos b = \sin b \cos z.$$

Since b and y are small, we may write,

$$y = b \cot z, \tag{50}$$

where y is in the same units as b, and the value of b is determined from Eq. (48) or (49). The equation shows that the correction is very small for objects near the horizon. When the observed star is near the north pole, we may substitute in this formula for the zenith distance z, $90^\circ - \phi$; hence we obtain,

$$y = b \tan \phi. \tag{51}$$

68. Chronometer.—Broadly speaking a chronometer is a large and well-constructed watch used principally in connection with the determination of longitude at sea. For this purpose it is supported in a ring by two bearings, and the ring is supported by two bearings at right angles to the first set, so that the chronometer remains horizontal whatever the inclination of the ship. A modern chronometer, with ordinary care on board ship, is capable of an accuracy in the variation of its rate of one second or less per 24 hours.

69. Instructions for Taking and Recording Observations.—Before beginning a determination, study carefully the directions and know what you have to do not only to begin but to finish your work. *Always have in mind the object of the determination and the quantities you have to measure.*

Prepare your field book to receive the measured quantities and have it as complete as possible. Never depend on your memory for any necessary data, no matter how obvious it may seem at the time. Assume that someone else at some future time will do the computing.

As a rule, observations include the time element. The observatory clock or chronometer is compared with the watch which is to be carried in the field, just before beginning and after finishing the observations. Before comparing the two, set the minute hand of the watch on a minute division mark when the second hand reads zero.

Focus the telescope with great care so as to secure a sharply defined image. Keep fingers off graduated circles. They

tarnish. Take time to level the instrument carefully, and do not disturb it during the observations. If the engineer's transit is used, it is a good plan to read and record the index error of the vertical circle. Usually the altitude or azimuth of a heavenly body is observed a number of times. If there is a doubt as to the correctness of an observation, reject it. Do not permit one poor observation to spoil a number of good ones.

When the engineer's transit is used, care must be exercised not to mistake the stadia wires for the middle horizontal wire.

When the *altitude* of a star is observed, point the telescope toward the star selected and get it in the field near the center. Clamp both vertical and horizontal circles, and, with the tangent screws, adjust so that the star will be crossing the *horizontal* wire near the vertical. After placing the horizontal wire a little ahead of the star, call to the recorder "Get ready." The instant the star is on the horizontal wire (very near the vertical one) call "Time." The recorder records the time of this observation and also the reading of the vertical circle, which the observer reads just after he calls "Time."

A similar procedure is followed in observing the sun. Observe the limb which is moving off the wire and call "Time" when tangency is reached. The point of tangency should be near the vertical wire.

The azimuth of a star or the sun is observed in a similar way. With the horizontal tangent screw, place the vertical wire a little ahead of the star and call to the recorder "Get ready"; the instant the star is on the *vertical* wire (very near the horizontal one) call "Time."

Read all horizontal angles clockwise. For the *direct* observations record the reading of vernier A, and for vernier B record what it actually reads minus 180°. For the *reverse* observations record the reading of vernier B, and for vernier A record what it actually reads minus 180°. This will facilitate averaging the whole set together.

It is well to plot the observations on cross-section paper before you begin to compute. Make the x-axis the time axis, and the y-axis the altitude. The plotted points should be on a straight line (Fig. 48) if there is no index error for the vertical circle. Otherwise the direct readings should be on a straight line parallel to another straight line through the reverse readings. When azimuth is observed, plot it on the y-axis. If the line of sight

of the instrument is adjusted all the points lie on a straight line, or two parallel lines, one for the direct and the other for the reverse readings. If the plotted points are scattered it is well to repeat the observations. If only one point is definitely off, reject it. Plot also the mean point by averaging all the times and corresponding altitudes or azimuths.

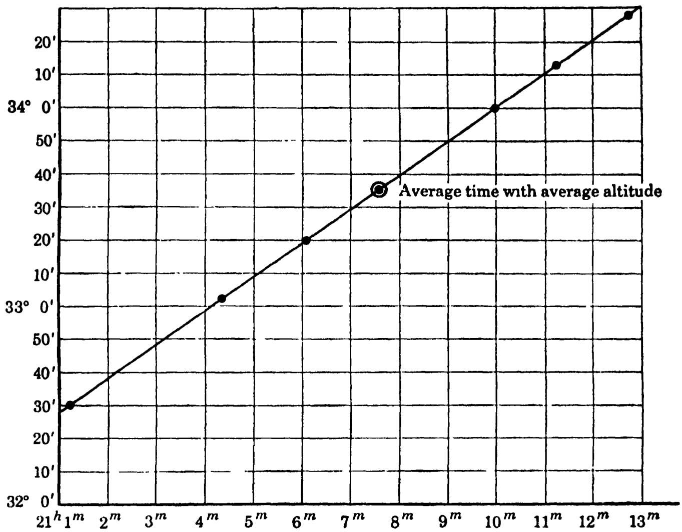

FIG. 48.—The altitudes of the star are plotted here against the corresponding times. All six observations lie on a straight line showing that no error was made in any one of them. If the index error of the engineer's transit were not equal to zero the three direct observations would lie on a line parallel to the line through the reversed observations.

Exercises

1. *a.* Find the index correction of a sextant given the following readings on the upper and lower limbs of the sun:

On the Arc °	′	″	Off the Arc °	′	″
0	36	50	359	33	10
	37	00		33	30
	36	50		33	30

b. From the above readings find the diameter of the sun.

2. To determine the error of runs of the reading microscopes of a theodolite, 10 measures of the 5′ divisions around the circle were made with the microscopes as follows:

Microscope A, average of 10 divisions: 4 revolutions 58.3 divisions of head.

Microscope B, average of 10 divisions: 4 revolutions 59.1 divisions of head.

The head of each microscope is divided into 60 divisions. The angle of 105° 12′ 17″.3 was read with microscope A; find its corrected reading.

3. Find the inclination of the axis of rotation of a transit instrument given the following level readings:

	Level Direct W	Level Direct E	Level Reversed W′	Level Reversed E′
First Reading:	50.5	10.9	50.8	90.3
Second Reading:	50.2	10.5	51.9	91.6

The value of one division of level = 0ˢ.057. Zero of level at one end of tube.

***4.** Find the correction due to the inclination of the axis of rotation of a theodolite when the azimuth of Polaris was measured at a latitude of 38° 50′ N. The following are the readings of the striding level.

	W	E
Direct:	14.1	11.2
Reversed:	14.8	11.9

The value of one division of level = 2″.5. Zero of level at middle of tube

CHAPTER VIII

DETERMINATION OF TIME

The most accurate means of determining time is by the transit instrument which will be taken up in Chap. XII. Here we shall consider methods suitable to the engineer's transit, the sextant, and the theodolite.

The problem of the determination of time consists in finding the error of the timepiece used at the instant of the astronomical observation. At a certain time according to the clock or chronometer used, an astronomical observation is made which yields the correct local time; this time compared with the reading of the clock will give its error. By the *correction of the clock we mean the amount to be added to the reading of the clock to obtain the correct time.* The correction is *positive* when the clock is slow and *negative* when fast.

When we are correcting a mean time clock we write

$$T = T' + \Delta T \tag{52}$$

where T' is the reading of the mean clock at the instant of the astronomical observation, ΔT is the clock correction, and T the correct mean time. Similarly, for a sidereal clock we have

$$\theta = \theta' + \Delta\theta. \tag{53}$$

The *rate* of a clock is the amount that it gains or loses per day; it is positive when it is losing and negative when it is gaining.

Astronomical observations yield *local time,* either apparent (T_a) or sidereal (θ). In case of apparent time, change it into civil by use of the equation of time ($T_a - T = E$).

The different methods of obtaining the local time may be grouped as follows:

I. *Time by meridian transits.*
II. *Time by equal altitudes.*
III. *Time by single altitudes.*

I. TIME BY MERIDIAN TRANSITS

$$\boxed{\theta = \alpha}$$

70. Time by Meridian Transits of Stars.—The *sidereal time* at the instant a star crosses the meridian of a place is equal to its right ascension ($\theta = \alpha$). If, therefore, an engineer's transit or a theodolite is set so that when rotated about its horizontal axis its line of sight describes the meridian, and if the reading of the sidereal clock is noted at the instant of the transit of a star of known right ascension, the clock correction will be given by the equation,

$$\Delta\theta = \alpha - \theta'.$$

The stars to be used in this determination must move rapidly, and hence only those near the equator should be chosen.

71. Observing List.—The table of Mean Places of Stars given in the American Ephemeris is consulted for stars suitable for this determination. For proper choice of stars the following points should be kept in mind:

a. Stars must be brighter than the fifth magnitude.

b. The difference in R.A. between stars to be observed must be greater than 4^m, to allow time between observations.

c. The declination of stars should be between $-30°$ and $+30°$, since rapidly moving stars will yield a better determination.

d. Stars with altitude greater than 55° cannot be seen with the engineer's transit. However, with the prismatic eyepiece, stars with altitudes up to 70° could be observed, although it is somewhat difficult to use the prismatic eyepiece, especially at night.

72. Preparing for Observations.—Let us consider the following example: With the engineer's transit find the correction to the sidereal clock on Jan. 10, 1930, in Cleveland ($\phi = 41°\ 32'\ N$ and $\lambda = 5^h\ 26^m\ W$). The latitude and longitude are known approximately; they may be obtained from some good map. The azimuth of a line at the place of observation is assumed to be known; from this azimuth the direction of the meridian is obtained. It is also assumed that the observations will commence at about 8:00 P.M., E.S.T. The local civil time corresponding to 8:00 P.M. is $12^h + 8^h - 26^m = 19^h\ 34^m$. To change this into sidereal time we use the approximate relation given in Eq. (20). Here $D = 111.1$ days and $T = 19^h\ 34^m$; this gives

$\theta = 26^h\ 52^m$, or the sidereal time corresponding to January 10, 8:00 P.M., E.S.T., is $2^h\ 52^m$. Stars to be observed are obtained from the list headed Mean Places of Ten-Day Stars in the American Ephemeris. Beginning with the R.A. of $2^h\ 52^m$ and having in mind the requirements set out above, the following observing list is prepared:

No.	Name	Magnitude	R.A.			Declination		Meridian altitude	
			h	m	s	°	′	°	′
1	η Eridani	4.05	2	53	0	− 9	11	39	17
2	α Ceti	2.82	2	58	37	+ 3	49	52	17
3	ζ Eridani	4.90	3	12	26	− 9	5	39	23
4	o Tauri	3.80	3	21	3	+ 8	47	57	15
5	f Tauri	4.28	3	27	0	+12	42	61	10

Notice that the stars selected are brighter than the fifth magnitude; the difference in R.A. is greater than 4^m; they are situated near the equator and no altitude is greater than 70°. The meridian altitude is obtained from the equation $h = 90° - z = 90° - (\phi - \delta)$.

73. Directions for Observing.

1. Level instrument over the station of azimuth line.
2. Point telescope to azimuth mark and clamp both horizontal plates.
3. Read horizontal circle.
4. Compute the circle reading for the south point from this reading and the known azimuth of the line. Set horizontal circle to this new reading; the instrument is now on the meridian.
5. Correct leveling in this position.
6. Set the vertical circle to the altitude of first star.
7. Call "Time" when star crosses *vertical wire.*
8. Reverse instrument and repeat process with the second star. Observe four stars.

In cases where the theodolite is used, the striding level is read direct and reversed after each star, to find the inclination of the axis of rotation. To correct the time of the meridian passage for each star for this inclination, use (from Art. 109) equation

$$t_b = b \cos (\phi - \delta) \sec \delta,$$

where t_b is the correction in seconds; b is the inclination of the axis (Art. 66); and δ is the declination of the star used. If the

azimuth of the line is known within 2″ and a theodolite is carefully oriented in the meridian, a clock correction determined by this method should be in error less than a second.

First-magnitude stars may be observed in daylight in a clear sky, if the instrument is in good focus. Great care must be exercised in focusing the instrument upon a distant terrestrial object if it is desired to make a determination in the daytime.

74. Computations.—The *apparent* right ascension of the stars observed is obtained from Apparent Places of Stars of the Ephemeris.

Example: In Cleveland on January 10, 1930, the following stars were observed with the engineer's transit:

Star	Apparent R.A.	Observed sidereal time of meridian passage	$\Delta\theta$
	h m s	h m s	s
η Eridani	2 53 0	2 53 6	− 6
α Ceti	2 58 37	2 58 38	− 1
ζ Eridani	3 12 26	3 12 31	− 5
o Tauri	3 21 3	3 21 13	−10
		Mean value of $\Delta\theta$ =	− 5.5.

Hence the clock is 5^s5 *fast.*

75. Time from Meridian Transit of the Sun.—In this case the meridian transits of the sun's western and eastern limbs are observed and the average of the two clock readings is obtained. This will be (with sufficient accuracy) the reading for the meridian transit of the sun's center. When the correction of the sidereal clock is desired, the apparent R.A. of the sun's center is obtained from the Ephemeris. When the correction of the mean clock is desired, the apparent time (12^h) is changed into mean (T) and the clock correction is then given by $\Delta T = T - T'$.

II. TIME BY EQUAL ALTITUDES

$$\boxed{\theta = \alpha}$$

76. Time by Equal Altitudes of a Star.—For any given place there are two positions, one on either side of the meridian, where a star has the same altitude. If, therefore, the sidereal-clock reading θ_1', when a star is east of the meridian, is noted,

and the reading θ_2' when the star has the *same altitude* west of the meridian, the average of θ_1' and θ_2' will give the clock reading when the star was on the meridian. Since the sidereal time at the instant a star is on the meridian is equal to its right ascension, we have for the correction of the sidereal clock

$$\Delta\theta = \alpha - \tfrac{1}{2}(\theta_1' + \theta_2').$$

If the correction of the civil clock is required, the R.A. of the star (*i.e.*, the sidereal time of the meridian passage) is changed into civil time (Art. 46) and the civil-clock correction will be given by

$$\Delta T = T - \tfrac{1}{2}(T_1' + T_2').$$

The advantage of the method is that neither the latitude of the place nor the declination of the star enters the determination, and the error of graduation of the instrument does not affect the accuracy of the determination. The disadvantages are the long wait between observations, and the possibility of clouds interfering with the second observation.

In the above equations the rate of the clock was not included; it may easily be allowed for, if necessary. The method is not suitable for observations on the sun, as its declination changes between the two observations, *so that the meridian passage does not occur half way between the instants of equal altitude.*

Directions for Observations.—Choose a star not near the meridian. The engineer's transit, the theodolite, or the sextant may be used; with the first two instruments care must be taken to set them on ground where no obstructions will interfere with the second reading. Place the star image in the field of view so that it is about to cross the horizontal wire near the vertical wire. *Clamp the vertical circle.* Record the time when the star crosses the horizontal wire. *Do not change the setting of the vertical circle.* For the second reading, if the instrument is not quite level, level it. Again record the time when the star crosses the horizontal wire near the vertical. The accuracy of the determination depends on the adjustments of the instrument and the care taken in leveling.

Example: Vega was observed at equal altitudes east and west of the meridian in Cleveland (long. $5^h\ 26^m\ 16^s$ W) on May 31, 1929. The readings of the civil chronometer were:

$$T'_1 = 22^h\ 18^m\ 10^s;\ T_2' = 29^h\ 36^m\ 4^s\ (5^h\ 36^m\ 4^s,\ \text{June 1}).$$

Determine the correction of the chronometer.

	h	m	s	Reference
Vega crossed the meridian of Cleveland at				
$\frac{1}{2}(T_1' + T_2')$ . . . . =	25	57	7	
That is, according to the chronometer, on June 1	1	57	7	
The Ephemeris gives the apparent R.A. of Vega				
for June 1. . . . . =	18	34	34	Art. 36
Changing this into mean time, we obtain. . T =	1	57	24	Art. 46
as the true local civil time of meridian passage.				
$\therefore \Delta T$ =			+ 17	Eq. (52)
Or the chronometer is 17^s slow.				

III. TIME BY SINGLE ALTITUDES

Given ϕ, z, and δ to find t

77. Time by Altitude of a Star.—The observation consists in noting the clock time θ' or T' and the corresponding altitude h' of a star near the prime vertical. Knowing the *altitude*, the *declination* of the star, and the *latitude* of the place, the three sides of the astronomical triangle are immediately obtained. From these the hour angle t of the star at the moment of observation is computed from Eq. (12). The R.A. of the star is obtained from the Ephemeris and $\theta = \alpha + t$ gives the true sidereal time of the observation. If the correction of the civil clock is required we change the sidereal time into civil ($\theta \rightarrow T$), and from $\Delta T = T - T'$ we obtain the clock correction.

78. Procedure when the Engineer's Transit or Theodolite Is Used.—Place the reflector over the objective for illuminating the cross-wires. Set the star image near the center of the field of the telescope and clamp both vertical and horizontal circles. With the tangent screws adjust so that the star will cross the *horizontal* wire near the vertical. Call "Time" to the recorder at the instant when the star is on the *horizontal wire* and near the vertical. Then the recorder puts down the observed time with the corresponding reading of the vertical circle. Three observations may be taken with the telescope direct, and three with the telescope reversed. If the readings are taken in close succession the average of the vertical angle readings and the average of the observed times are used in the computation. It is sometimes impossible to reverse the instrument. In this case, to avoid instrumental errors, two stars may be observed, one about due east and the other about due west; the average of the

two clock corrections will be nearly free from instrumental errors.

Schedule of Observations

1. Take three observations on star with telescope direct, reading watch and vertical circle each time.
2. Take three observations on star with telescope reversed, reading watch and vertical circle.

Outline for Reduction of Observations

	Reference
1. Plot on cross-section paper the observed "times" against the corresponding "altitudes" to see if linear relations hold.	Art. 69
2. Average watch time and obtain average clock time (T').	
3. Obtain average altitude (h').	
4. The average zenith distance $z' = 90° - h'$.	
5. Correct z' for refraction, $z = z' + r$.	Eq. (32)
6. Obtain from Ephemeris apparent place of the star.	Art. 36
7. Compute t and reduce it to time units.	Eq. (12) and Table I, or Eq. (9) when calculating machine is used.
8. Obtain sidereal time (θ) from $\theta = \alpha + t$.	
9. Change θ into local civil time (T).	Art. 46
10. Clock correction $\Delta T = T - T'$.	

Example: To find the time from single altitudes of a star with the engineer's transit.

Warner and Swasey Observatory (lat. 41° 32′ 13″ N; long. $5^h\ 26^m\ 16^s.4$ W).

Date: June 3, 1929. *Star:* Vega.
Transit: No. 2. *Position:* East of meridian.

Observations

Telescope	Watch h	m	s	Vertical circle °	′	″
Direct	21	1	10	32	30	0
		4	23	33	2	0
		6	8	33	20	0
Reversed		10	4	34	0	0
		11	14	34	12	0
		12	46	34	28	0

Reading of	Before observing			After observing		
	h	m	s	h	m	s
Clock (local civil)	20	28	0	20	49	0
Watch....	20	54	28	21	15	28

Reductions

Figure 48 shows that there are no particular irregularities in the observations.

	h	m	s
Average watch time	21	7	37.5
Correction, watch to clock		−26	28 0
Average clock time T'.	20	41	9.5
Average altitude h'	33°	35′	20″

	°	′	″		
$h' =$	33	35	20		
$z' =$	56	24	40		
$r =$		+1	31	$\log \sin \frac{1}{2}[z + (\phi - \delta)]$	= 9 694076
$\phi =$	41	32	13	$\log \sin \frac{1}{2}[z - (\phi - \delta)]$	= 9.654155
$\delta =$	38	42	48	$\log \sec \frac{1}{2}[z + (\phi + \delta)]$	= 0 432922
$\phi - \delta =$	2	49	25	$\log \sec \frac{1}{2}[z - (\phi + \delta)]$	= 0.009446
$z =$	56	26	11		
$\phi + \delta =$	80	15	01	$\log \tan^2 \frac{1}{2}t$	= 9.790599
				$\log \tan \frac{1}{2}t$	= 9.895300
$z + (\phi - \delta) =$	59	15	36	$\frac{1}{2}t$	= 38° 9′ 34″
$z - (\phi - \delta) =$	53	36	46	t	= 76° 19′ 08″
$z + (\phi + \delta) =$	136	41	12		h m s
$z - (\phi + \delta) =$	−23	48	50	t	= −5 5 16.5
$\frac{1}{2}[z + (\phi - \delta)] =$	29	37	48	α of Vega	= 18 34 33.8
$\frac{1}{2}[z - (\phi - \delta)] =$	26	48	23	$\theta = \alpha + t$	= 13 29 17.3
$\frac{1}{2}[z + (\phi + \delta)] =$	68	20	36	$(\theta \to T)$* T	= 20 41 9.2
$\frac{1}{2}[z - (\phi + \delta)] =$	−11	54	25	T'	= 20 41 9.5
				ΔT	= − 0.3

	h	m	s
* Sidereal time of Greenwich, 0^h, June 3..	16	43	50 6
Reduction to Cleveland meridian.........			+53.6
Sidereal time of Cleveland, 0^h, June 3	16	44	44.2
Sidereal time of observation, (θ)..........	13	29	17.3
Sidereal interval after 0^h................	20	44	33.1
Reduction to mean time interval.........		3	23.9
Local mean time (T)....................	20	41	9.2

79. Time by Altitude of a Star When Sextant Is Used.—When the observations are made at sea the instrument is held in the right hand and the telescope is pointed at the horizon. The image of the star received from the index mirror is brought to the horizon. Slightly rotate the sextant about the axis of the telescope (by twisting the wrist) so that the star will appear to describe an arc. When this arc is tangent to the horizon call "Time." The observation must be made early in the evening or morning, or during bright moonlight, to enable the observer to see the horizon.

If the observation is made on land, the artificial horizon is used. The two images (from the mercury and from the index mirror) are moving with respect to each other; separate them in such a direction that they appear to be coming together, and at the instant they come together call "Time."

It will be found convenient to keep the image reflected from index mirror oscillating slightly by twisting the wrist, and to call "Time" when its arc crosses the image of the star from the mercury. *It is never good policy to make observations while moving the tangent screw.*

The following suggestions are for land observations with the artificial horizon:

1. Take at least four readings on any conveniently located star to determine the index error of the sextant (Art. 61).
2. Observe the star to be used for the time determination and read angle and corresponding watch time. Make at least two such readings.
3. Reverse the glass roof of the artificial horizon.
4. Observe the same star again, as in 2.

Outline of Computations

1. Plot on cross-section paper the observed "times" against the corresponding "double altitudes" to see if linear relation holds.
2. Average watch readings and obtain average clock reading (T').
3. Average double altitudes (R).
4. Correct for index error using $2h' = R + i$ (Art. 61).
5. $z' = 90° - h'$

The rest is the same as in Art. 78.

Example: To find the correction to the civil clock from single altitudes of a star, with the sextant.

Warner and Swasey Observatory (lat. 41° 32′ 13″N; long. $5^h\ 26^m\ 16^s.4$ W).

Date: June 5, 1929. *Star:* Vega.

Sextant: No. 5. *Position:* East of meridian.

OBSERVATIONS

Reading of	Before			After		
	h	m	s	h	m	s
Clock	20	5	0	20	24	0
Watch	20	31	36	20	50	36
Correction, watch to clock =		−26	36		−26	36

Index Observations

	′	″		′	″
1.	5	40	3.	5	30
2.	5	40	4.	5	30

Roof	Double altitude			Watch reading		
	°	′	″	h	m	s
A	60	10	10	20	39	2
	61	16	30		42	21
B	61	47	40		43	53
	62	24	10		45	43

	h	m	s
Average watch reading	20	42	45
Watch correction..............		−26	36
Average clock reading (local civil), T' = ...	20	16	9

		°	′	″
Average of double altitudes,	R =	61	24	38
Index correction......	i =		− 5	35
	$2h'$ =	61	19	3
	h' =	30	39	32
	z' =	59	20	28

Completion of this example is left to the student.

80. Time by Altitude of the Sun.—The observation consists in noting the clock time (T' or θ') and the corresponding altitude (h') of the sun. Since we then know the *altitude* and the *latitude* of the place and the *declination* of the sun at the *approximate time* of observation, the three sides of the astronomical triangle are determined. As will be explained in Art. 81, it is better to make the observation when the sun is away from the meridian. The hour angle (t) is found by solving this triangle, Eq. (12), and from it the apparent time is obtained ($T_a = 12^h + t$). The equation of time (E) for the *approximate time* of observation,

applied to T_a, will give the local civil time (T) of the observation ($E = T_a - T$), from which the correction of the clock may be obtained.

Procedure When Engineer's Transit Is Used.—Use shade glass to diminish intensity of the sun's light or project image of the sun and crosswires on a piece of paper as described in Art. 57.

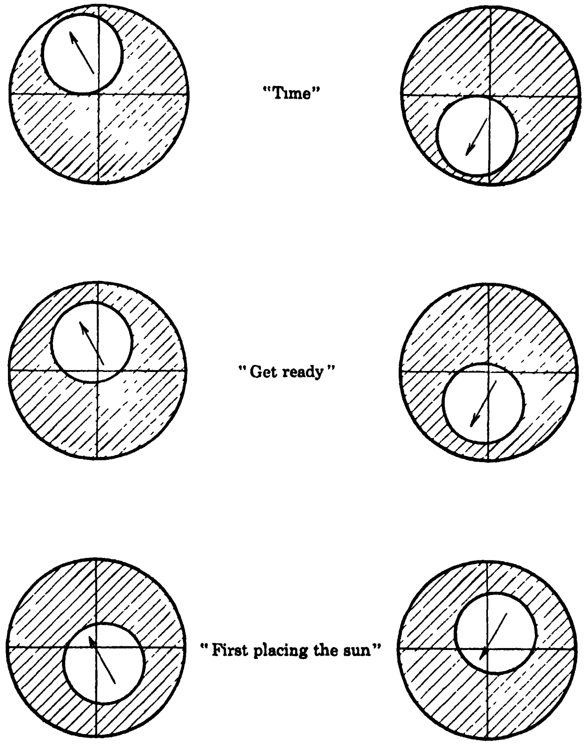

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FIG. 49. FIG. 50.

FIGS. 49 and 50.—Observing the sun with the telescope and prismatic eyepiece *inverting*. The *lower* limb is observed in the morning and the *upper* in the afternoon.

Place the sun's limb in such a position that it will be *leaving* the horizontal wire (Figs. 49 and 50). Clamp both motions, and focus accurately the combined image of sun and cross-wires of transit. The instant the sun's disk becomes tangent to the horizontal wire call "Time." The recorder puts down the watch reading with the corresponding reading of the vertical circle. Secure three readings direct and three reversed. The

average of the altitudes is to be corrected for refraction, parallax, and semidiameter.

OUTLINE FOR REDUCTION OF OBSERVATIONS

	Reference
1. Plot observations on cross-section paper, altitude against time.	Art. 69
2. Average watch readings and obtain clock reading (T').	
3. Average altitudes (h').	
4. $z' = 90° - h'$.	
5. $z = z' + 60''.6 \tan z' - 8''.8 \sin z' \pm S$.	Chap. VI
6. Obtain declination (δ) of sun from Ephemeris with the known approximate time of observation.	Art. 34
7. Compute hour angle (t) and reduce it to time units.	Eq. (12) and Table I, or Eq. (9) when calculating machine is used.
8. Secure from Ephemeris the equation of time (E) with the known approximate time of observation.	Art. 34
9. $T = 12^h + t - E$.	
10. Clock correction $\Delta T = T - T'$.	

If the approximate time, assumed for the purpose of finding the declination of the sun and the equation of time, differs by more than 3^m from the value found in step 9, a recomputation is necessary. This is made by first getting a new value of the declination and equation of time, using the improved approximate time of the observation just found.

Procedure When Sextant Is Used.—The method is the same as that used for a star (Art. 79). The index correction is obtained by observing the upper and lower limbs of the sun as has been explained in Art. 61. The double altitude for both the upper and the lower limb of the sun is observed, and the average yields the double altitude of its center.

Example: Find the correction of the local civil clock of the Warner and Swasey Observatory from single altitude of the sun with the sextant.

Lat. 41° 32′ 13″ N; long. $5^h\ 26^m\ 16^s.4$ W.

OBSERVATIONS

Date: June 18, 1929, *Sextant*: No. 3,

Reading of	Before Observations			After Observations		
	h	m	s	h	m	s
Clock	15	59	00	16	43	00
Watch	16	24	30	17	8	30

Correction of watch to clock $-25^m\ 30^s.0$

Observations for Index Error

	On the Arc			Off the Arc		
	°	′	″	°	′	″
1.	0	29	10	359	46	50
2.	0	28	40	359	46	00

No.	Lower limb Roof A						Upper limb Roof B					
	Watch reading			Double altitude			Watch reading			Double altitude		
	h	m	s	°	′	″	h	m	s	°	′	″
1.	16	56	13	63	16	50	17	0	37	62	39	40
2.	16	59	1	62	14	10	17	1	49	62	13	20

REDUCTION

	h	m	s
Average watch reading.............	16	59	25.0
Correction of watch to clock........		−25	30.0
Average clock reading T'...........	16	33	55 0
	°	′	″
Average double altitude R..........	62	36	00
$R_1 =$	0	28	55
$R_2 =$	359	46	25

	°	′	″				
R	62	36	00	$\log\sin\frac{1}{2}[z + (\phi - \delta)]$	9.793710		
i		−7	40	$\log\sin\frac{1}{2}[z - (\phi - \delta)]$	9.540983		
$2h'$	62	28	20	$\log\sec\frac{1}{2}[z + (\phi + \delta)]$	0 326582		
h'	31	14	10	$\log\sec\frac{1}{2}[z - (\phi + \delta)]$	0.000629		
z'	58	45	50				
r		+1	40	$\log\tan^2\frac{1}{2}t$	9.661904		
p			−8	$\log\tan\frac{1}{2}t$	9.830952		
z	58	47	22				
ϕ	41	32	13	$\frac{1}{2}t$	34°	7′	13″
δ	23	25	08	t	68°	14′	26″
					h	m	s
$\phi - \delta$	18	7	05	or t	4	32	57.7
$\phi + \delta$	64	57	21	$T_a = 12^h + t$	16	32	57.7
				E			−55.4
				Local civil time (T)...	16	33	53.1
$z + (\phi - \delta)$	76	54	27	Clock time of observation (T')............	16	33	55.0
$z - (\phi - \delta)$	40	40	17	Clock correction (ΔT)..			−1.9
$z + (\phi + \delta)$	123	44	43				
$z - (\phi + \delta)$	−6	9	59				
$\frac{1}{2}[z + (\phi - \delta)]$	38	27	14				
$\frac{1}{2}[z - (\phi - \delta)]$	20	20	09				
$\frac{1}{2}[z + (\phi + \delta)]$	61	52	22				
$\frac{1}{2}[z - (\phi + \delta)]$	−3	5	00				

81. Position of Heavenly Body Most Favorable for Determination of Time by Single Altitude.—It has been indicated above that the best position of a star or the sun for the determination of time by single altitude is near the prime vertical. This can be shown as follows: Differentiate Eq. (9)

$$\sin h = \sin \phi \sin \delta + \cos \phi \cos \delta \cos t$$

considering h and t as variables; we obtain

$$\cos h \cdot dh = -\cos \phi \cos \delta \sin t \cdot dt.$$

That is

$$dt = \frac{-\cos h}{\cos \phi \cos \delta \sin t} dh.$$

Since from the law of sines

$$\frac{\cos h}{\cos \delta} = \frac{\sin t}{\sin A},$$

we have on substitution,

$$dt = \frac{-1}{\cos \phi \sin A} dh$$

From this we see that the error dt in the computed hour angle is numerically a minimum for a given error dh in the observed altitude, when the azimuth A is 90° *or* 270°.

Exercises

1. Prepare an observing list of four stars for determining the sidereal time by meridian transits of stars with the engineer's transit, assuming the direction of the meridian given. The latitude of the place is 40° 10′ N and the longitude $5^h\ 50^m$ W. The observations are to be made at about 10 P.M., C.S.T., on June 5, 1930.

2. In Cleveland (long. $5^h\ 26^m\ 16^s$ W) on Aug. 30, 1930, Altair was observed at equal altitudes at the following readings of the E.S.T. clock:

	h	m	s
East of meridian	7	56	19 P. M.
West of meridian	11	22	37 P. M.

Find the clock correction assuming the rate of the clock to be zero.

3. Complete the example given in Art. 79.

4. Find the correction of the civil clock of Cleveland (lat. 41° 32′ 13″ N; long. $5^h\ 26^m\ 16^s.4$ W) from the following altitude measures on the sun's upper limb, made with an engineer's transit on June 17, 1929.

Telescope	Watch reading			Vertical circle		
	h	m	s	°	′	″
Direct	16	24	26 5	38	10	00
		25	41.0	37	55	40
		27	8 5	37	39	00
Reversed	16	29	51.5	37	9	00
		30	56.0	36	58	40
		31	36.5	36	51	20

Reading of	Before observing			After observing		
	h	m	s	h	m	s
Clock	15	53	00.0	16	8	00.0
Watch	16	19	35.0	16	34	35.0

CHAPTER IX

LATITUDE

The methods for determining the astronomical latitude of a place may be grouped as follows:

I. *By meridian altitude.*
II. *By circummeridian altitudes.*
III. *By altitude of a star not on the meridian, when the time is known.*
IV. *By the zenith telescope.*

The first three methods yield approximate determinations and are best adapted to the engineer's transit, the sextant, or the theodolite. In fact, the second will yield fairly precise results with the theodolite. The fourth method is by far the most precise; it will be described in Chap. XIV.

Since the latitude of a place may be defined as the altitude of the pole, the average of the altitudes (corrected for refraction and index) at the upper and lower culminations of a circumpolar star will give the latitude. This, of course, requires a wait of 12 hours and for obvious reasons is not usually a suitable method for determination.

I. LATITUDE BY MERIDIAN ALTITUDE

This method is based essentially on Eqs. (1), (2), and (3), that is:

$z_m = \phi - \delta$ for upper transits south of the zenith.
$z_m = \delta - \phi$ for upper transits north of the zenith.
$z_m = 180° - (\phi + \delta)$ for lower transits of circumpolar stars.

Knowing the declination of the heavenly body and observing the meridian altitude or zenith distance, we obtain the latitude.

82. Latitude by Transit of Polaris.—Twice in 24 sidereal hours Polaris is on the meridian of the observer, hence it can usually be observed at either upper or lower transit. Since it describes a small circle, only a little over one degree in radius about the pole, its altitude change is very small during an interval of a few minutes before or after transit. For this reason it is suitable

for determining latitude with instruments of the precision of the sextant, engineer's transit, or even the theodolite. Also, it is easily identified, as it is a relatively bright star (second magnitude) with no other bright star near it.

Since the pole, Polaris, and δ Cassiopeiæ, in the order given are approximately in a straight line, the two stars will be at

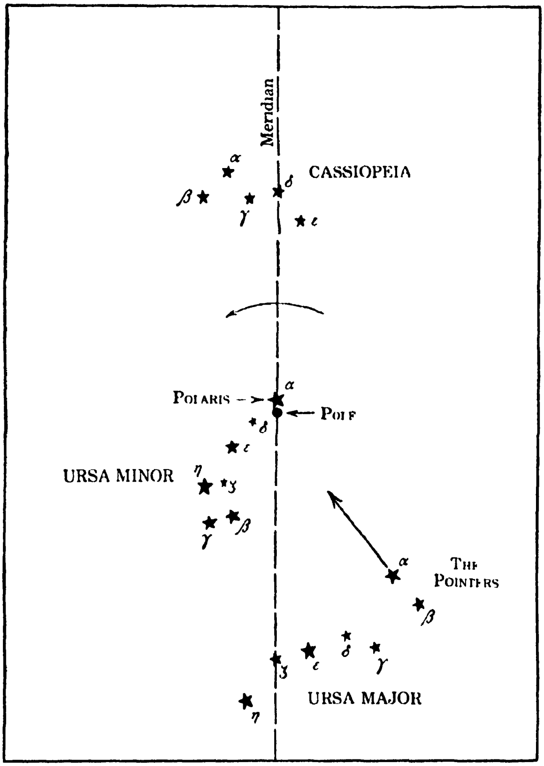


FIG. 51.—Principal stars near the north celestial pole. Showing Polaris and δ Cassiopeiæ at upper culmination and ζ Ursæ Majoris at lower culmination.

upper or lower culmination approximately at the same time (Fig. 51). This enables one to tell by the appearance of the sky when Polaris is approaching a culmination. The apparent place and the civil time of upper culmination of Polaris may be obtained from Table VII of the American Ephemeris. The time of lower culmination is 12 sidereal hours later or $11^h\ 58^m\ 2^s$ later in mean time reckoning.

In the evening, long before Polaris is visible with the unaided eye, it may be observed with the engineer's transit if the approximate latitude is known. With this and the declination of Polaris its meridian altitude is obtained; if the vertical circle is set to read this altitude and the telescope swung in the direction of the north, Polaris may be easily found. Care must be taken to have the telescope well focused on a distant object before attempting to see the star under these conditions.

If the engineer's transit or the theodolite is used, follow Polaris for a few minutes before culmination, using the tangent screw of the vertical circle, until it has reached the highest or lowest altitude and appears to move on the horizontal wire. This is the meridian altitude from which the meridian zenith distance is obtained and when corrected for refraction is substituted in Eq. (2) if at upper transit or in Eq. (3) if at lower. The apparent declination of Polaris is obtained either from Table VII of the Ephemeris or from the Table of Apparent Places of Circumpolar Stars. A number of other tables give the declination and other useful data for Polaris; the "Ephemeris of the Sun and Polaris" is very convenient (see p. 41.)

Example: In Cleveland (long. 5^h 26^m W), on June 4, 1929, Polaris was observed at upper culmination for determining the latitude.

	°	′	″
Observed altitude	42	38	30
Index error			00
Altitude	42	38	30
Zenith distance (z_m')	47	21	30
Refraction correction (Table IV)		+1	03
Corrected zenith distance (z_m). .	47	22	33
Declination of Polaris (δ) . .	88	55	13
Latitude (ϕ).	41	32	40

When the time is known within a minute it is well to take one observation with telescope direct, a minute or more before culmination, then reverse the instrument and take another observation. In the latitudes of the United States, the altitude of Polaris near culmination remains practically unchanged for about eight minutes. The average of the two altitudes yields a value free from index or collimation error. Other circumpolar stars may be observed and the latitude computed in exactly the same manner.

83. Latitude from Meridian Transit of Stars.—A star attains its greatest altitude at the instant it crosses the upper meridian. Hence, if we observe it continuously and record altitudes with the corresponding times for about fifteen minutes before it crosses the meridian and continue observing for about the same

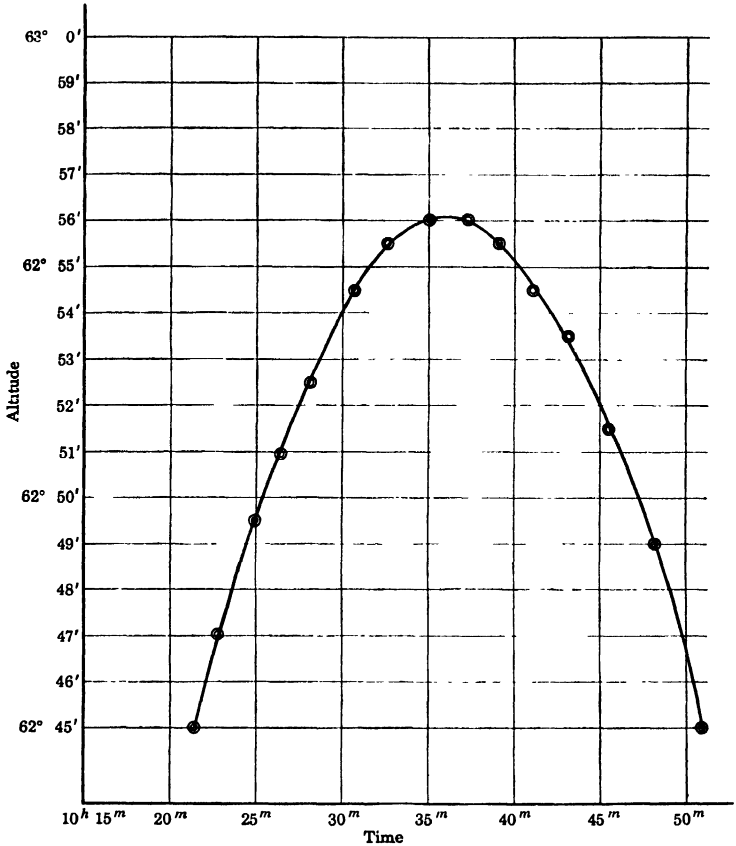


FIG. 52.—Change in altitude of α Herculis while crossing the meridian.

time after it has crossed the meridian, the highest altitude recorded is approximately the meridian altitude. This, corrected for index error and refraction and substituted in Eq. (1) or (2), will yield the latitude. The time corresponding to the greatest altitude will be the time of meridian transit, hence a rough correction of the clock may also be obtained.

Inasmuch as more than one star is to be observed, an observing list is necessary. To prepare such a list, follow the directions

given in Art. 71. (Of course here it is not necessary to use fast moving stars.) For the latitude necessary in preparing the list, use the best approximate value known.

If all the observations for each star are plotted in rectangular coordinates, using time for abscissa and altitude for ordinate, the resulting smooth curve is approximately a parabola. Hence, if a parabola is drawn as nearly as possible through the plotted points, a better value for the meridian altitude and time may be obtained (Fig. 52). Have in mind that the longest ordinate divides the parabola symmetrically.

Example: In Cleveland (long. $5^h\ 26^m\ 16^s.4$ W) on July 7, 1930, α Herculis was observed near the meridian with the engineer's transit. The altitudes and the corresponding times were both recorded in order to determine the latitude of the place and the correction of the clock.

Eastern Standard Time	Altitude	Eastern Standard Time	Altitude
h m s	° ′ ″	h m s	° ′ ″
10 21 36 P.M.	62 45 00	10 35 05 P.M.	62 56 00
23 01	47 00	37 32	56 00
24 50	49 30	39 08	55 30
26 39	51 00	41 23	54 30
28 23	52 30	43 05	53 30
30 40	54 30	45 36	51 30
32 39	55 30	47 56	49 00
		51 03	45 00

Figure 52 shows these observations plotted and a parabola drawn through the points.

REDUCTION

	° ′ ″	Reference
Estimated highest altitude	62 56 10	Fig. 52
Zenith distance (z'_m)	27 3 50	
Refraction (r)	+29	Table IV
Corrected zenith distance (z_m)	27 4 19	
Apparent declination of star from Eph. (δ)	14 28 4	Art. 36
Latitude, $\phi = z_m + \delta$	41 32 23	
	h m s	
Apparent R.A. of α Herculis from Eph. (α)	17 11 29 5	Art. 36
Local sidereal time (θ)	17 11 29 5	$\alpha = \theta$
Change $\theta \rightarrow T$, July 7, 1930	22 10 0 8	Art. 46
Difference in longitude between Cleveland and 75° meridian	+26 16 4	
True Eastern Standard Time of transit	22 36 17 2	
Observed time of transit	22 36 10 0	Fig. 52
Clock correction (ΔT)	+7 2	

***84. Latitude by Circummeridian Altitudes.**—The observations given in the above illustration are usually referred to as circummeridian altitudes and can be used for a still more precise determination of latitude by reducing each altitude to the meridian. If the time is known, the result is nearly as accurate as if all the observations were made on the meridian, provided they are taken sufficiently close to it, say not more than 15 minutes away. Equations (1) and (2) may be combined by considering, for the northern hemisphere, the north meridian zenith distance as negative; that is, the equation

$$\phi = \delta + z_m \tag{54}$$

holds for stars both north and south of the zenith. For stars off the meridian a correction must be applied to their zenith distance z, to reduce it to z_m.

From trigonometry, we have

$$\cos t = 1 - 2 \sin^2 \tfrac{1}{2}t.$$

Substituting this Eq. (9), we obtain

$$\cos z = \sin \phi \sin \delta + \cos \phi \cos \delta - 2 \cos \phi \cos \delta \sin^2 \tfrac{1}{2}t,$$

or

$$\cos z = \cos (\phi - \delta) - 2 \cos \phi \cos \delta \sin^2 \tfrac{1}{2}t,$$

or by Eq. (54)

$$\cos z_m - \cos z = 2 \cos \phi \cos \delta \sin^2 \tfrac{1}{2}t.$$

From trigonometry, we have

$$\cos z_m - \cos z = -2 \sin \tfrac{1}{2}(z_m + z) \sin \tfrac{1}{2}(z_m - z).$$

Hence,

$$\sin \tfrac{1}{2}(z_m + z) \sin \tfrac{1}{2}(z_m - z) = -\cos \phi \cos \delta \sin^2 \tfrac{1}{2}t. \tag{55}$$

Since the observations are to be made within 15 minutes on either side of the meridian, z_m and z are very nearly equal, and we may write

$$\sin \tfrac{1}{2}(z_m + z) = \sin z$$
$$\sin \tfrac{1}{2}(z_m - z) = \tfrac{1}{2}(z_m - z) \sin 1'',$$

since $\sin x = x'' \sin 1''$, when x is very small.

Hence Eq. (55) takes the form

$$(z_m - z) \sin 1'' \sin z = -2 \cos \phi \cos \delta \sin^2 \tfrac{1}{2}t$$

or

$$z_m = z - \cos \phi \cos \delta \csc z \frac{2 \sin^2 \frac{1}{2}t}{\sin 1''}. \tag{56}$$

The value of the hour angle t for each observation is obtained from $\theta = \alpha + t$. If the civil time (T) is used when observing, change the right ascension of the star into civil time, T_0 (Art. 46), and find $T - T_0$. Change this civil-time interval into sidereal reckoning; the result is the hour angle (t) for the star at the instant of observation. An approximate value of ϕ is required; this may be obtained by the method of Art. 82 or of Art. 83; or it may be already known. If the computed value of ϕ differs materially from the assumed one, a recomputation is necessary using this computed value instead of the original in Eq. (56).

$$\text{Let } A = \cos\phi \cos\delta \csc z \text{ and } m = \frac{2\sin^2\frac{1}{2}t}{\sin 1''},$$

then Eq. (56) becomes

$$z_m = z - A \cdot m. \tag{57}$$

To facilitate the work of computing, tables have been prepared for A and m (see Chauvenet's "Manual of Spherical and Practical Astronomy," or other works of similar nature). Table VI gives values of m.

Example: Using the data of the example in Art. 83, and assuming the correction of the clock $\Delta T = -2^s.8$, and the latitude as 41° 32′ N, we have:

Zenith distance, z			t		A	m	$A \cdot m$	Reduced meridian zenith distance, z_m		
°	′	″	m	s		″	″	°	′	″
27	15	29	14	46	1.58	428.01	676	27	4	13
	13	29	13	21	1.58	359.84	569			00
	10	59	11	32	1.58	261.12	413			06
	9	29	9	43	1.58	185.35	293			36
	7	59	7	58	1.58	124.61	197			42
	5	59	5	41	1.59	63.42	101			18
	4	59	3	42	1.59	26.88	43			16
	4	29	1	15	1.59	3.07	5			24
	4	29	1	12	1.59	2.83	4			25
	4	59	2	48	1.59	15.39	24			35
	5	59	5	04	1.59	50.40	80			39
	6	59	6	46	1.59	89.89	143			36
	8	59	9	18	1.58	169.80	268			31
	11	29	11	38	1.58	265.68	420			29
	15	29	14	45	1.58	427.04	675			14

	°	′	″
Average of z_m =	27	4	24
Apparent declination of α Herculis, δ =	14	28	04
$\phi = \delta + z_m$ =	41	32	28

No recomputation is necessary as this value agrees well with the one assumed.

Column 1. The zenith distance is obtained from the observed altitude and corrected for refraction by means of Table IV.

Column 2. The true standard time of the transit of α Herculis over the Cleveland meridian was computed in the example of Art. 83. It is $T_0 = 22^h\,36^m\,17^s.2$. To each observed time, the known clock correction of $-2^s.8$ is applied; for example, for the first observation, $T_1 = 22^h\,21^m\,36^s - 2^s.8 = 22^h\,21^m\,33^s.2$. Then, disregarding sign $T_1 - T_0 = 14^m\,44^s$; on reducing this into sidereal interval, we have $t = 14^m\,46^s$.

Column 3. The value of $A = \cos\phi\cos\delta\csc z$ may be computed directly or it may be obtained from tables. The approximate value for the latitude is used.

Column 4. The value of m is given in Table VI.

Column 6. The reduced meridian zenith distance is $z_m = z - A \cdot m$ [Eq. (57)].

The method described above furnishes a good determination of latitude (correct to about 2″), if the theodolite is employed and the time known within one second. In such a case, the following additional refinements must be considered:

1. To avoid instrumental errors, observe one star with telescope direct, the next with telescope reversed, and so on.

2. Observe the same number of stars with about the same zenith distances north and south, to eliminate errors due to refraction.

85. Latitude from Meridian Transit of Sun.—The method of observing is exactly the same as for a star (Art. 83). The sun's lower limb is observed. No appreciable error is introduced by assuming the declination constant during the duration of observations, as its rate of change is always less than 1′ per hour. It is convenient to know the approximate time of transit so as to know when to commence observing. This may be found by changing 12^h apparent time into civil time (Art. 44), assuming that the latter is the time available. Begin observing 10 or 12 minutes before the sun crosses the meridian, and continue for about the same time after the crossing. The highest altitude, corrected for semidiameter, refraction, and parallax, will be the approximate meridian altitude. Or, as has been explained in the case of a star, all the observations may be plotted and the meridian altitude and civil time of transit chosen from the figure.

Example:

OBSERVATIONS OF SUN FOR LATITUDE DETERMINATION

Cleveland (long. $5^h\ 26^m\ 16^s$ W) *Engineer's transit:* No. 4 *Index error* = 00″	*Date:* July 7, 1930 *Limb observed:* lower *Known clock correction* = -3^s

No.	Eastern Standard Time			Altitude		
	h	m	s	°	′	″
1	12	23	49	70	45	30
2		24	46		46	30
3		25	50		47	30
4		26	55		48	00
5		28	01		48	30
6		28	50		49	00
7		29	53		49	00
8		31	36		49	00
9		33	07		48	30
10		33	59		48	00
11		35	04		48	00
12		35	57		47	00
13		36	54		47	00
14		37	51		46	00

	°	′	″
Observed maximum altitude (h')	70	49	00
Zenith distance (z')	19	11	00
Semidiameter (S).		−15	46
Refraction (r). .		+	21
Parallax (p) . .		−	3
Corrected zenith distance (z_m) . .	18	55	32
Declination of the sun for apparent noon (δ) .	22	37	23
Latitude $\phi = \delta + z_m$ =	41	32	55

*The method of circummeridian altitudes (Art. 84) may be employed here for a more precise determination. The observed times are corrected by the known clock correction and changed into civil time (T). By use of the equation of time, T is changed into apparent time (T_a). From $T_a = 12^h + t$, the hour angle (t) is secured which is needed for the reduction of the zenith

distances to the meridian [Eq. (56) or (57)]. From the above example, we have

Zenith distance, z			t		A	m	$A \cdot m$	Zenith distance reduced to meridian, z_m		
°	′	″	m	s		″	″	°	′	″
18	59	02	7	09	2 12	100 4	212	18	55	30
	58	02	6	12	2 12	75.5	160			22
	57	02	5	08	2 12	51.7	110			12
	56	32	4	03	2.12	32.2	68			24
	56	02	2	57	2.12	17.1	36			26
	55	32	2	08	2.12	8.9	19			13
	55	32	1	05	2.12	2.3	5			27
	55	32	0	38	2 12	0.8	2			30
	56	02	2	09	2.12	9.1	19			43
	56	32	3	01	2.12	17.9	38			54
	56	32	4	06	2 12	33.0	70			22
	57	32	4	59	2.12	48.8	103			49
	57	32	5	56	2 12	69 1	146			06
	58	32	6	53	2.12	93 0	197			15

	°	′	″
Average of z_m .	18	55	27
Declination of the sun for apparent noon (δ)	22	37	23
Latitude, $\phi = \delta + z_m =$	41	32	50

Column 1. The zenith distance is obtained from the observed altitude and is corrected for semidiameter, refraction (Table IV), and parallax (Table V).

Column 2. For example, the Eastern Standard Time given for the first observation is $12^h\ 23^m\ 49^s$. Applying to this the known clock correction of -3^s and the difference in longitude of $26^m\ 16^s$, we have for the local civil time of the observation $T = 11^h\ 57^m\ 30^s$. The equation of time for this instant is $-4^m\ 39^s$. Hence the corresponding apparent time $T_a = 11^h\ 52^m\ 51^s$ and therefore $t = T_a - 12^h = -7^m\ 09^s$.

Column 3. The value of $A = \cos\phi \cos\delta \csc z$ may be computed directly [see Eq. (56)], or obtained from tables. The approximate value of 41° 32′ is used for the latitude and the declination is taken from the Ephemeris for the instant of each observation.

Column 4. m is obtained from Table VI.

Column 6. The reduced meridian zenith distance is $z_m = z - A \cdot m$ [Eq. (57)].

II. LATITUDE BY ALTITUDE OF A STAR WHEN TIME IS KNOWN

Given t, δ, and z to find ϕ

86. This method is based on observing the altitude of a star (preferably Polaris) and the corresponding time. From the altitude, the zenith distance z is obtained; from the time and the right ascension of the star, its hour angle t. The declination δ is taken from the Ephemeris. We therefore have two sides z and $90° - \delta$ of the astronomical triangle and the angle t, from which we may solve for the other side $90° - \phi$, and hence find the latitude. From the law of cosines, we have Eq. (9):

$$\cos z = \sin \phi \sin \delta + \cos \phi \cos \delta \cos t$$

with ϕ the only unknown. To secure a suitable formula for its solution we make the following two assumptions

$$\sin \delta = f \cos F \tag{58}$$

$$\cos \delta \cos t = f \sin F \tag{59}$$

where f and F are auxiliary unknowns. They give, when substituted in Eq. (9),

$$\begin{aligned} \cos z &= f \cos F \sin \phi + f \sin F \cos \phi, \\ &= f \sin (F + \phi). \end{aligned}$$

Substituting the value of f from Eq. (58) in the last equation, we get

$$\sin (F + \phi) = \cos F \cos z \csc \delta. \tag{60}$$

Dividing Eq. (59) by Eq. (58), we have

$$\tan F = \cot \delta \cos t. \tag{61}$$

Equation (61) gives the value of F which is taken in the first or fourth quadrant according to the algebraic sign of the tangent; and Eq. (60) gives the value of $(F + \phi)$, and hence, ϕ. The proper quadrant for $(F + \phi)$ is chosen from knowledge of F and the approximate latitude.

Schedule of Observations for engineer's transit and theodolite:

1. Take three or more readings of the altitude of Polaris and the corresponding time.
2. Reverse telescope and take the same number of readings.

Outline of Reduction of Observations

	Reference
1. Average watch readings and obtain average civil time (T').	
2. Apply the known clock correction (ΔT) to T' to obtain the local civil time (T).	$T = T' + \Delta T$
3. Change T to sidereal time (θ).	Art. 45
4. Average altitudes (h') and obtain z'.	$z' = 90° - h'$
5. $z = z' + r$.	Table IV
6. Obtain the apparent place of Polaris at the time of observation.	Art. 35
7. Find t from equation $\theta = \alpha + t$ and change it into angular units.	Table I
8. Compute F from Eq. (61), and ϕ from Eq. (60).	

Table I of the American Ephemeris, which gives the difference in altitude between Polaris and the pole, may be used here to advantage if a shorter but less accurate computation is desired. Compute t as above, and with it and the declination of Polaris secure from this table the quantity to be added to or subtracted from the *corrected* average altitude to get the latitude. A similar table is to be found in the "Ephemeris of the Sun and Polaris" (see Art. 33).

Example: In Cleveland on June 11, 1929, the following altitudes of Polaris were measured with the engineer's transit. It is required to find the latitude.

Telescope	Watch time	Vertical circle
	h m s	° ′ ″
Direct....	21 28 18	40 29 30
	30 58	29 30
	32 33	29 30
Reversed .	36 38	30 00
	38 00	30 00
	39 25	30 00

	h m s
Average of watch readings.................	21 34 19
Known watch and clock correction........ ..	−29 23
Local civil time (T).............	21 4 56
Average altitude (h')......	40° 29′ 45″

Coordinates of Polaris: $\begin{cases} \alpha = 1^h\ 35^m\ 31^s. \\ \delta = 88°\ 55'\ 13''. \end{cases}$

Reduction

	h	m	s
$\theta =$	14	24	40
$\alpha =$	1	35	31
$t =$	12	49	9
	°	′	″
$t =$	192	17	15
$h' =$	40	29	45
$z' =$	49	30	15
$r =$		+ 1	8
$z =$	49	31	23

$$\tan F = \cot \delta \cos t$$

$$\cot \delta = 0.018847$$
$$\cos t = -0.977092$$
$$\tan F = -0.018415$$

$$\sin (\phi + F) = \cos F \cos z \csc \delta$$

$$\cos F = 0.999830$$
$$\cos z = 0.649142$$
$$\csc \delta = 1.000178$$

$$\sin (\phi + F) = 0.649148$$

	°	′	″
$\phi + F =$	40	28	39
$F =$	−1	3	18
$\phi =$	41	31	57 N

REDUCTION BY MEANS OF TABLE I OF EPHEMERIS

We have from Ephemeris,

88° 55′ 10″		88° 55′ 20″		δ / t	
′	″	′	″	h	m
63	16	63	6	12	51
63	27	63	17	12	48

For $t = 12^h\ 49^m.15$ we have the interpolated values 63′ 23″ and 63′ 13″, so that for $\delta = 88°\ 55'\ 13''$ we have, on interpolating again, 63′ 20″.

	°	′	″
Corrected altitude.	40	28	37
From interpolation.	+	63	20
Latitude.	41	31	57

Table I of A. E. has been computed for altitude of 45°. For other altitudes, corrections taken from Table Ia, A. E., may be applied when greater accuracy is required.

87. Selection of Stars.—It has been pointed out above, that Polaris is preferred for latitude determinations. It will be shown here that any other star near the meridian is suitable.

Differentiating Eq. (9) considering ϕ and z as variables, we have

$$-\sin z \cdot dz = (\sin \delta \cos \phi - \cos \delta \cos t \sin \phi)\, d\phi.$$

The expression in the parenthesis is $-\sin z \cos A$ by Eq. (11): hence, substituting and solving for $d\phi$, we have

$$d\phi = \sec A \cdot dz,$$

from which we see that a small error dz in *measuring the zenith distance, or altitude, will have the least effect in the determination of latitude when the azimuth* (A) *is* 0° *or* 180°.

Differentiating Eq. (9), considering now ϕ and t as variables and making the same substitution [Eq. (11)] we have,

$$\sin z \cos A \cdot d\phi + \cos \phi \cos \delta \sin t \cdot dt = 0$$

or

$$d\phi = -\frac{\sin t \cos \delta}{\sin z \cos A} \cos \phi \cdot dt.$$

Since

$$\sin t \cos \delta = \sin z \sin A \text{ [Eq. (8)], we have}$$

$$d\phi = -\tan A \cos \phi \cdot dt,$$

from which we see, that *a small error* (dt) *in the assumed time will have the least effect in the determination of latitude when the azimuth is* 0° *or* 180°.

88. Latitude from Polaris without the Ephemeris, the Time Being Known. (From Comstock's "Field Astronomy").—If Polaris were exactly at the pole, its altitude corrected for refraction would be the latitude of the place, and its azimuth would always be 180°. Since Polaris describes a small circle of about one degree radius about the pole, some correction must be applied to its altitude and azimuth to reduce them to the pole. These corrections are given in Table VII for the year 1930 and for latitude 40° N. That is, for the year 1930 and for latitude 40° N, the altitude and azimuth for Polaris will be $A = 180° + a$, and $h = \phi + b$. The table naturally shows a negative when Polaris is seen west of the meridian, zero when at the meridian, and positive when seen east; b is positive when Polaris is above the pole; and negative, below. *The refraction is included in the value of* b. The hour angle of Polaris (t), which is an argument in this table, is obtained from $\theta = \alpha + t$. The given time of observation, which is assumed to be known with an accuracy not necessa-

rily greater than one minute, is changed into sidereal time (θ) by Eq. (20), and the right ascension of Polaris is given in Table IX.

Since the declination of Polaris varies from year to year, chiefly on account of the precession of the equinoxes (Art. 31), the values of a and b will change with time. Also these values will differ for different latitudes; hence, we may write for the coordinates of Polaris for any year and at any latitude,

$$A = 180° + F_1 a \tag{62}$$

$$h = \phi + F_2 b \tag{63}$$

where F_1 and F_2 are factors used to modify the values of a and b of Table VII; they are given in Tables VIII and IX, respectively. The results obtained by this method may in some cases be more than one minute of arc in error.

Example: Without the Ephemeris, compute the azimuth of Polaris for 1931, September $29^d\ 20^h\ 6^m$ local civil time of Cleveland, and also the latitude of the place if the observed altitude is $41°\ 48'$.

To change the given local civil time into sidereal without the Ephemeris we use Eq. (20), $\theta = T + 4^m \cdot (1 - \frac{1}{70}) \cdot D$; $T = 20^h\ 6^m$ and $D = 29^d.8 - 21^d.7 = 8^d.1$; hence, $\theta = 20^h\ 38^m$.

From Table IX we obtain $\alpha = 1^h\ 38^m$ and $t = \theta - \alpha = 19^h\ 0^m$.

Table VII gives $a = +81'$ and $b = +17'$, while Tables VIII and IX give, respectively, $F_1 = 1.03$ and $F_2 = 1.01$. Substituting these values into Eqs. (62) and (63), we have

$$A = 180° + 81' \cdot 1.03 = 181°\ 23'.$$
$$h = \phi + 17' \cdot 1.01.$$

or

$$\phi = h - 17' = 41°\ 31'.$$

Exercises

1. On June 3, 1929, in Cleveland, Polaris was observed at lower transit with the engineer's transit as follows:

		°	'
Altitude	Telescope direct	40	28
	Telescope reversed	40	29

Find the latitude of the place.

2. In Cleveland, on July 15, 1930, the following circummeridian observations were made with the engineer's transit on α Ophiuchi:

Observations

No.	Eastern Standard Time			Altitude		
	h	m	s	°	′	″
1	10	15	28	61	0	00
2		16	26		1	00
3		16	52		1	30
4		17	35		2	00
5		20	29		3	30
6		22	43		4	30
7		24	39		5	00
8		26	32		5	00
9		28	23		4	30
10		30	41		3	30
11		32	44		2	30
12		34	59		0	30
13		36	13	60	59	30

The index error of the transit is 0″.

Plot the observations and obtain the meridian altitude of the star, and hence the latitude. Also find the correction of the chronometer.

Apparent place of α Ophiuchi from Ephemeris: $\begin{cases} \alpha = 17^h\ 31^m\ 43^s.3 \\ \delta = 12^\circ\ 36'\ 31''. \end{cases}$

Longitude of Cleveland $= 5^h\ 26^m\ 16^s$W.

***3.** Reduce the circummeridian observations of Exercise 2 to the meridian and obtain the latitude, assuming the correction of the chronometer to Eastern Standard Time to be -23^s.

4. The following circummeridian altitudes of the sun's lower limb were observed on July 2, 1930, in Cleveland (long. $5^h\ 26^m\ 16^s$ W) with the engineer's transit:

Eastern Standard Time			Altitude		
h	m	s	°	′	″
12	18	9	71	6	00
	20	2		8	30
	22	16		11	00
	24	11		13	00
	27	46		15	00
	29	12		15	00
	32	57		15	00
	35	27		14	00
	37	50		12	30
	40	05		10	00
	41	27		8	30

Index error $= 00''$.

Plot the observations and find the highest altitude, and from it obtain the latitude. Also obtain the correction of the chronometer used.

5. On June 14, 1929, in Cleveland, the altitude of Polaris and the corresponding time were observed in the manner explained in Art. 86. The average uncorrected altitude is 40° 27′ 57″ and the average corrected time is $20^h\ 34^m\ 42^s.4$. Determine the latitude.

CHAPTER X

AZIMUTH

89. General Principle.—One of the astronomical determinations most important to the civil engineer and surveyor is that of azimuth. *The azimuth of a line may be defined as the horizontal angle which the line makes with the north and south line, measured clockwise and usually from the south point.* Knowing the azimuth of a line, the direction of the meridian may be laid out on the ground by simply setting off an angle equal to the azimuth of the line and in the counterclockwise direction from it.

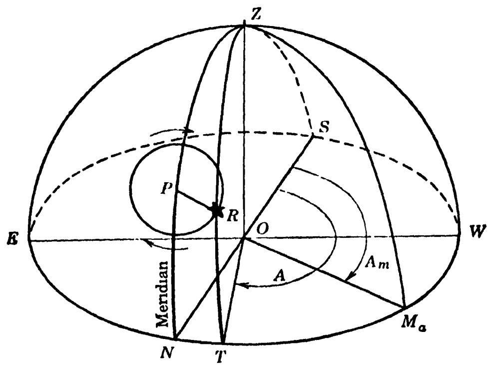


Fig. 53.

If the North Star were exactly over the north pole, the horizontal angle formed at the instrument between a distant signal mark and the North Star would give at once the azimuth of the line joining the instrument and the mark. This, of course, is not the case. If, however, the deviation of the North Star from the north line is known for the time of observation, the azimuth of the line can easily be found. Broadly speaking then, the problem of finding the azimuth of a line may be divided into two parts: first, the measuring of the horizontal angle between the azimuth line and the heavenly body; and, second, finding the azimuth of the heavenly body. Figure 53 illustrates this. The azimuth of the line OM_a is required. The observer at O measures the

horizontal angle between the azimuth mark M_a and the star R (angle M_aOT). He records the time of sighting on the star, or reads the altitude on the vertical circle of his transit; from this and other data the angle SOT is computed. By subtracting from angle SOT the angle M_aOT, he obtains the azimuth of the line.

Let A_m = the azimuth of a line measured clockwise from the south point.

A = the azimuth of the heavenly body measured in the same manner.

M = horizontal circle reading on the mark.

K = circle reading on the heavenly body, obtained by releasing the *upper plate*, sighting on the heavenly body and reading the angle in the *clockwise direction*.

$$K - M = \text{angle } M_aOT.$$

Then we may write

$$\boxed{A_m = A - (K - M).} \tag{64}$$

This equation holds for all positions of the azimuth mark and star, provided the angles are read in the *clockwise direction*, and the azimuth is *reckoned clockwise from the south point.*

This chapter deals primarily with the methods of observing circumpolar stars and the sun together with different methods of computing their azimuth (A).

90. Azimuth Marks.—The end of the azimuth line on which the instrument is set is called the *station*, the other, the *azimuth mark*. It is well to have the mark set as far away from the station as possible so that no refocusing is necessary in changing from mark to star. In the daytime, a pole will be found suitable, or a box painted with black and white stripes and accurately lined with a more or less permanent ground mark (stake, or stone, or concrete monument). At night, a lantern placed in a box with a hole bored in the front of the box will be found suitable. The size of the hole and the brightness of the lamp depend on the distance from the station. The ideal arrangement is to make the azimuth mark and star look nearly alike. A 10-watt lamp without any covering, placed at a distance of a mile and a half, has been found quite satisfactory by the writer.

91. Azimuth by Circumpolar Stars at Elongation.—There are two points on the diurnal circle of every circumpolar star at which the star appears farthest east and farthest west. These points are called the *points of elongation.* They are points of tangency with two vertical circles (Fig. 54); hence the triangles PRZ and $PR'Z$ have right angles at R and R', respectively.

The *time of eastern or western elongation of a star* is obtained by computing the angle at the pole in the astronomical triangle, as follows:

Angle at the star R or $R' = 90°$.

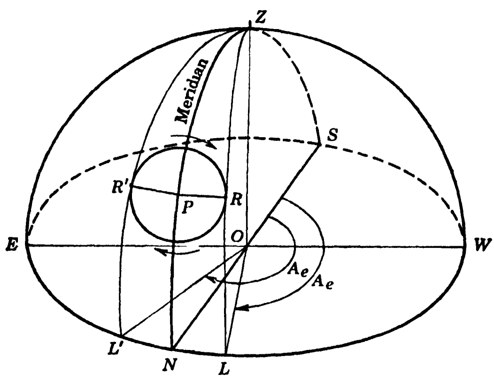


FIG. 54.—A circumpolar star at eastern elongation R' and at western elongation R. The vertical circles ZL and ZL' are tangent to the diurnal circle of the star at R and R'.

Angle at the pole $P = 360° - t_e$ (eastern elongation) or t_e (western).

Arc $PZ = 90° - \phi$.

Arc PR or $PR' = 90° - \delta$.

From the relation of the angles of a spherical triangle and its three sides [Eq. (6)], we have for both cases:

$$\sin z \cos 90° = \sin (90° - \delta) \cos (90° - \phi) - \cos (90° - \delta) \sin (90° - \phi) \cos t_e$$

or

$$\cos \delta \sin \phi - \sin \delta \cos \phi \cos t_e = 0.$$

Solving for $\cos t_e$, we have

$$\cos t_e = \cot \delta \tan \phi. \qquad (65)$$

For western elongation, t_e is in the first quadrant; and for eastern, in the fourth. The latitude (ϕ) is assumed to be known and the coordinates of the star (α, δ) are secured from the Ephemeris. From $\theta = \alpha + t_e$, the sidereal time of the elongation is obtained, which may be changed to civil.

Example: Find the local civil time of eastern elongation of Polaris in Cleveland ($\lambda = 5^h\, 26^m\, 16^s\!.4$ W, $\phi = 41°\, 32'\, 13''$ N) for July 25, 1930.

	Reference
Coordinates of Polaris, July 26.0 $\left\{ \begin{array}{l} \alpha = 1^h\, 36^m\, 50^s\!.0 \\ \delta = 88°\, 55'\, 34.0 \end{array} \right\}$	Art. 35
Hour angle of Polaris at eastern elongation, $t_e = 270°\, 57'\, 05''$.	Eq. 65
t_e (in time) $= 18^h\ 3^m\, 48^s\!3$.	Table I
Sidereal time of eastern elongation, $\theta = 19\ 40\ 38.3$.	$\theta = \alpha + t_e$
Civil time of eastern elongation, $T = 23\ 27\ 58.8$.	Art. 46

Table VII of the Ephemeris gives the civil time of elongation of Polaris. Making use of this table to obtain the above, we have:

	h	m	s
Civil time of upper culmination, meridian of Greenwich, July 26, 1930	5	24	05
To reduce it to the Cleveland meridian, interpolate for $5^h\, 26^m$, *i.e.*, $5.44 \cdot (-9^s\!78)$			−53
Civil time of upper culmination, meridian of Cleveland	5	23	12
To reduce this to the civil time of eastern elongation, subtract (last column, Table VII, for lat. 41° 32′ N)	5	55	15
Cleveland Civil Time of eastern elongation	23	27	57

The azimuth at the instant of eastern or of western elongation of a star is obtained from the law of sines for the astronomical triangle (Fig. 54), *i.e.*,

$$\text{For western elongation: } \frac{\sin\,(180° - A_e)}{\sin\,(90° - \delta)} = \frac{\sin 90°}{\sin\,(90° - \phi)}.$$

$$\text{For eastern elongation: } \frac{\sin\,(A_e - 180°)}{\sin\,(90° - \delta)} = \frac{\sin 90°}{\sin\,(90° - \phi)}.$$

Hence,

$$\sin A_e = \pm \cos\delta \sec\phi, \tag{66}$$

where the sign must be chosen so that, for western elongation, A_e is in the second quadrant; and for eastern, in the third. The declination of the star is obtained from the Ephemeris and the latitude from a map (it is not necessary to know it with great

accuracy) or from a previous determination. Table V of the Ephemeris gives the azimuth of Polaris at elongation as does Table X of this book.

The civil time of elongation of Polaris and its azimuth are also given in the "Ephemeris of the Sun and Polaris" published by the U. S. Department of Interior, General Land Office.

92. Procedure for Determining Azimuth by Observing Polaris at Elongation, the Time Being Known.

	Reference
1. Compute time of elongation.	Art. 91
2. Carefully set and level instrument over station.	
3. Read horizontal circle when telescope is pointing on mark.	M of Eq. (64)
4. A few minutes before elongation, release upper plate and point on Polaris.	
5. Read horizontal circle clockwise with Polaris on *vertical wire* near horizontal, at the instant of elongation.	K of Eq. (64)
6. If the theodolite is used, read striding level direct and reversed.	Art. 66
7. Read on mark as in 3.	

The above directions may be slightly modified to increase the accuracy by eliminating instrumental errors as follows:

In view of the fact that the azimuth of Polaris from about four minutes before to four minutes after elongation, differs from its azimuth at elongation by less than one second of arc (in latitudes of the United States), it is clear that the time need not be accurately known. As a matter of fact two sets of observations may be made during this time; one with telescope direct, and the other with telescope reversed.

Directions for Computing

	Reference
1. Obtain coordinates of Polaris from Ephemeris or other source.	Art. 35
2. Compute the azimuth at elongation, or secure it from Ephemeris; for western elongation, A_e is in the second quadrant; and for eastern elongation, in the third.	Eq. (66) or Table V, A.E.
3. Average the readings on the mark to obtain M of Eq. (64).	
4. K = average horizontal angle reading on Polaris.	
5. If the striding level is used add to K, $b \tan \phi$.	Art. 67
6. The required azimuth of the mark will be $A = A_e - (K - M)$.	Eq. (64)

***93. Azimuth of Circumpolar Star near Elongation.**—To increase the accuracy of the determination of azimuth at elongation, observations may be taken before and after the time of elongation, and the azimuth of the star in these positions reduced to the azimuth at elongation. This has the effect of increasing the number of readings at elongation, just as in the determination of latitude, circummeridian altitudes were observed and reduced to the meridian.

The reduction is effected by the following equation, which was taken from Campbell's "Practical Astronomy":

$$A_e - A = \pm\frac{\sin\delta\cos\delta}{\sin z_e}\cdot\frac{2\sin^2\frac{1}{2}(t_e - t)}{\sin 1''} \tag{67}$$

where $A_e - A$ is the correction to be applied to the horizontal circle reading of the star when its hour angle is t. The upper sign is for eastern elongation, δ is the declination of the star, and t_e the hour angle of the star at elongation. The zenith distance at elongation, z_e, is computed from

$$\sin z_e = \cos\phi\sin t_e. \tag{68}$$

(Law of sines for the right spherical triangle *PRZ*.)
If for convenience, we replace the last fraction of Eq. (67) by m, we have

$$A_e - A = \pm\frac{\sin\delta\cos\delta}{\sin z_e}\cdot m.$$

The value of $t_e - t$ is equivalent to the sidereal time difference between the time of elongation and the observed time; the value of m may be obtained directly from Table VI. To shorten the work, let the average of the values of m be denoted by m_0; then the last equation becomes

$$A_e - A = \pm\frac{\sin\delta\cos\delta}{\sin z_e}\cdot m_0. \tag{69}$$

This correction is now applied to the average horizontal reading on the star instead of correcting each reading and averaging the results.

Schedule of Observations

1. Compute time of elongation.
2. Carefully set and level instrument over station.
3. Read horizontal circle when telescope is pointing on mark.

4. About 15 minutes before elongation, release upper plate and point toward star.
5. Read striding level direct and reversed.
6. Call "Time" when star crosses vertical wire near horizontal and record the horizontal angle and the corresponding time.
7. Make three or more such pointings on the star.
8. Read striding level as in 5.
9. Point on mark.
10. Reverse telescope and repeat process.

The required azimuth of the mark is given by Eq. (64),

$$A_m = A_e - (K - M).$$

The time and azimuth at elongation are computed as before (Art. 91). The value of M is the average of the four readings on the mark. To the average of the readings on the star, the following corrections are applied to obtain K: (*a*) $A_e - A$ as explained above, (*b*) for level [Eq. (51)], (*c*) for aberration (Art. 96), and (*d*) for error of runs of microscopes (Art. 65). The time

Station: "New" *Lat.* 41° 32′ 13″.1 N *Long.* 5^h 26^m $16^s.4$ W	*Theodolite:* Troughton and Simmis *Striding level:* d = 2″.5. *Sidereal chronometer correction:* $+9^s$	*Star:* Polaris at eastern elongation *Date:* July 25, 1930

No.	Sight	Sidereal chron. reading	Horizontal angle	Reading micr. *A*	Reading micr. *B*
			Telescope Direct		
		h m s	° ′	′ ″	′ ″
1	Mark	....	110 10	4 40 2	4 44.0
2	Read level				
3	Star	19 20 34	144 10	0 48 9	0 54.0
4	Star	19 29 20	144 10	0 55 8	1 02.1
5	Star	19 31 57	144 10	1 00.7	1 06.9
6	Read level				
7	Mark	....	110 10	4 43.0	4 48.1
			Telescope Reversed		
8	Mark	....	110 10	4 45.0	4 48.0
9	Read level				
10	Star	19 44 48	144 10	1 08.8	1 15.6
11	Star	19 48 20	144 10	1 08.8	1 15.6
12	Star	19 51 41	144 10	1 00.6	1 06 9
13	Star	19 58 29	144 10	0 52.5	0 57.0
14	Read level				
15	Mark	....	110 10	4 48.0	4 54.0

must be known with an accuracy of three or four seconds, and if a sidereal timepiece is not available, the intervals $T_e - T$ should be changed into sidereal intervals. When the engineer's transit is employed it is not, of course, necessary to apply corrections (*b*), (*c*), and (*d*), the time need not be known more accurately than within a minute, and the timepiece need not keep sidereal time.

When observing Polaris, reduction to elongation may be further simplified by use of Table XI. First compute the azimuth at elongation A_e (in this table the azimuth is referred to the north point), and with this and $t_e - t$ obtain the correction $A_e - A$ from this table. Table V_a of the American Ephemeris is more extensive.

Other stars well adapted for observation are δ and λ Ursæ Minoris and 51 Cephei (Fig. 56).

To illustrate the above a complete determination is given beginning on page 133.

LEVEL READINGS

No.	Direct		Reversed	
	W	E	W′	E′
2	1	22	12	11
6	0	23	12	11
9	2	21	16	7
14	0	23	18	5

Zero of level at center of tube.

REDUCTION

	Reference
The time of eastern elongation, $\theta_e = 19^h\ 40^m\ 38^s$.	Computed in Art. 91
$\sin A_e = \pm \cos \delta \sec \phi$.	Eq. (66)
Azimuth of eastern elongation, $A_e = 181°\ 26'\ 05''.2$.	
$\sin z_e = \cos \phi \sin t_e$.	Eq. (68)
$t_e = 270°\ 57'\ 05''$.	Computed in Art. 91
$\sin z_e = +0.748425$	

No.	Circle reading			$\theta_e - \theta$		m
	°	′	″	m	s	″
3	144	10	51.4	19	55	778.4
4		10	59.0	11	09	244.1
5		11	03.8	8	32	143 0
10		11	12.2	−4	19	36 6
11		11	12 2	−7	51	121.0
12		11	03.8	−11	12	246.3
13		10	54.8	−18	00	635.9

Average = 144° 11′ 02″.5 $m_0 = 315''.0$

The circle readings are for the average the two reading microscopes. $\theta_e - \theta = t_e - t$ is the sidereal time interval between elongation and the instant of observation. The values of m are obtained from Table VI.

$$\sin\delta = 0.999824.$$
$$\cos\delta = 0.018742.$$

		Reference
(a)	$A_e - A = +\frac{\sin\delta\cos\delta}{\sin z_e}\cdot m_0 = +7''.9.$	Eq. (69) plus "for eastern elongation."
(b)		

	(2)	(6)	(9)	(14)	Mean
$b =$	$-12''.5$	$-13''.8$	$-6''.3$	$-6''.3$	$-9''.7$

Eq. (48)

$y = -9''.7\cdot\tan\phi = -8''.6.$ — Eq. (51)

(c) Aberration $= +0''.31\cos A = -0''.3.$ — Art. 96

$K = 144°\ 11'\ 2''.5 + 7''.9 - 8''.6 - 0''.3 = 144°\ 11'\ 01''.5.$

$M = 110°\ 14'\ 46''.3.$ — Average of 1, 7, 8, and 15.

$A_m = A_e - (K - M) = 147°\ 29'\ 50''.0.$ — Eq. (64).

94. Azimuth by a Circumpolar Star at Any Hour Angle, the Time being Known.—This method is based on the solution of the astronomical triangle, (Fig. 53). After the circle reading on the azimuth mark (M) is recorded, the instrument is turned toward the star, and the horizontal circle reading (K) and corresponding time (T' or θ') obtained. The coordinates of the star, α and δ, are secured from the Ephemeris. The clock correction and latitude are assumed known (Chaps. VIII and IX). Apply to T' or θ' the clock correction and obtain the true sidereal time (θ) of the observation. $\theta = \alpha + t$ then gives the hour angle (t).

From triangle PZR, we have

$$PZ = 90° - \phi \text{ and } PR = 90° - \delta.$$

Angle at $P = t$, when the star is west of meridian, and $360° - t$ when east of the meridian. The angle at Z is required and is equal to $180° - A$ or $A - 180°$, according as the star is west or east of the meridian.

To derive a suitable equation for computing the azimuth (A), divide Eq. (8) by Eq. (11),

$$\tan A = \frac{\sin t\cos\delta}{\sin\phi\cos\delta\cos t - \cos\phi\sin\delta}. \tag{70}$$

Another expression may be obtained by dividing the numerator and denominator of Eq. (70) by $\cos\phi\sin\delta$, *i.e.*,

$$\tan A = \frac{\sin t \cot\delta \sec\phi}{\tan\phi\cot\delta\cos t - 1}. \tag{71}$$

Let

$$a = \tan\phi\cot\delta\cos t. \tag{72}$$

Then we have

$$\tan A = -\sin t\cot\delta\sec\phi\cdot\frac{1}{1-a}. \tag{73}$$

The advantage of this equation over Eq. (70) is that tables have been prepared for finding $\log\frac{1}{1-a}$ directly from $\log a$, when $\log a$ is computed from Eq. (72).

Schedule for Observing

1. Set instrument over station and level carefully.
2. Point to mark and record circle reading.
3. Release upper plate and point to star; read circle (clockwise) and time when star crosses vertical wire near horizontal.
4. Repeat 3.
5. Point to mark and record circle reading.
6. Reverse telescope and repeat process.

To illustrate the method of observing and computing, the following determination will be treated in full:

Observations

Station: Howe	Date: June 23, 1930	Reading of	Before	After
			h m s	h m s
Lat.: 41° 32′ 13″ N	*Star:* Polaris	*Watch*	23 22 42	23 39 41
Long.: $5^h\ 26^m\ 16^s$ W	*Transit:* No. 4	*Local civil clock*	22 58 00	23 15 00

Sight	Watch time	Vernier *A*	Vernier *B*
	Telescope Direct		
	h m s	° ′ ″	° ′ ″
Mark	....	0 0 0	0 0 0
Star	23 35 00	23 31 45	23 32 00
Star	23 37 10	23 32 15	23 32 45
Mark	....	0 0 0	0 0 0
	Telescope Reversed		
Mark	....	0 0 0	0 0 0
Star	23 27 20	23 30 15	23 30 30
Star	23 30 00	23 31 00	23 31 15
Mark	....	0 0 0	0 0 0

Correction, watch to clock $= -24^m\ 41^s.5$.
Known civil clock correction $= +11^s.4$.

REDUCTION

		Reference
	h m s	
Average watch time	23 32 22.5	
Watch correction	−24 41.5	
Civil clock time	23 7 41.0	
Civil clock correction	+11.4	Given
Local civil time of observation (T)	23 7 52.4	
Corresponding sidereal time (θ)	17 14 18 8	Art. 45
α of Polaris	1 36 12.1	Art. 35
t of Polaris	15 38 06.7	$\theta = \alpha + t$
	° ′ ″	
t of Polaris (in arc)	234 31 41	Table I
δ of Polaris	88 55 32	Art. 35
Latitude (ϕ)	41 32 13	Given

$$\tan A = \frac{\sin t \cot \delta \sec \phi}{\tan \phi \cot \delta \cos t - 1} \qquad \text{Eq. (71)}$$

$\sin t = -0.814400$	$\tan \phi = 0.885875$
$\cot \delta = 0.018754$	$\cot \delta = 0.018754$
$\sec \phi = 1.335955$	$\cos t = -0.580304$

$$\tan A = \frac{-0.020404}{-0.009641 - 1} = 0.020209;$$

	° ′ ″	
A =	181 9 28	A is in the third quadrant since $t = 15^h\ 38^m$
Average circle reading on star (K) =	23 31 28	
Average circle reading on mark (M) =	0 0 0	
Azimuth of line (A_m) =	157 38 00	$A_m = A - (K - M)$

Figure 55 shows the observed and computed angles.

95. Some Approximate Methods for Computing the Azimuth of Stars.—1. *The American Ephemeris* gives in Table IV the azimuth of Polaris at all hour angles. This table may be used when a value of the azimuth is required correct to about 6″. The latitude of the station and the hour angle of Polaris are the arguments in the table; the azimuth from the north point is obtained by double interpolation (Art. 37).

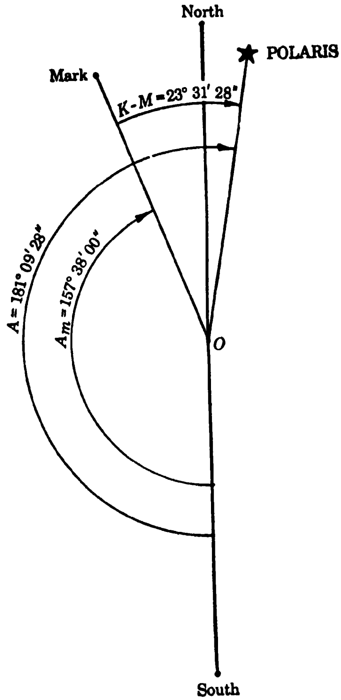


FIG. 55. $A_m = A - (K - M)$.

Example: Consider the data given above:

$$t = 15^h\ 38^m\ 06\overset{s}{.}7 \text{ and } \phi = 41°\ 32'\ 13''.$$

Table IV, American Ephemeris, gives:

40°		42°	Lat. / H.A.
° ′		° ′	h m
1 7.9		1 10.0	15 40
1 7 5	1° 9′1	1 9 6	
1 5.7		1 7.7	15 30

The underlined numbers are computed by interpolation.

$$A = 180° + 1°\ 9'1 = 181°\ 9'1.$$

Table IV has been computed for a declination of Polaris of 88° 55′ 55″ (year 1930). For other declinations the correction given in Table IV*a* should be applied to the azimuth taken from Table

IV. This correction in the case given above is equal to $+ 0'.4$; hence, we have

$$A = 181° 9'.1 + 0'.4 = 181° 9'.5.$$

2. *The Ephemeris of the Sun and Polaris*, published by General Land Office, gives the azimuth of Polaris for mean-time hour angles instead of sidereal hour angles, as given in Table IV of the American Ephemeris. It also gives, for every day in the year, the civil time of the upper transit of Polaris for the Greenwich meridian; for any other meridian a simple interpolation is necessary. Having thus the local civil time of the upper transit of Polaris for the local meridian and the local civil time of the observation the mean-hour angle of Polaris is obtained.

In the example given above we have:

		h	m
Local civil time of observation	$T =$	23	7.9
June 23, 1930, local civil time of upper transit of Polaris. . . .	$=$	7	32.3
Mean-time hour angle of Polaris.	$t_m =$	15	35.6

The interpolation for the azimuth is the same as in the work given above.

3. *When the latitude of the station is not known*, the altitude of the star at the beginning and the end of each set of readings on the star is recorded. The average of the readings, changed to zenith distance and corrected for refraction, will yield the true zenith distance z. Then Eq. (8) gives the azimuth

$$\sin A = \frac{\sin t \cos \delta}{\sin z}. \tag{74}$$

Example: The average altitude in the illustration of Art. 94, is 40° 55′ 30″; hence, z is 49° 4′ 30″ + 1′ 6″ = 49° 5′ 36″. As computed above, $t =$ 234° 31′ 41″ and δ = 88° 55′ 32″. Hence, using natural functions, we have

$$\sin A = \frac{-0.814400 \cdot 0.018751}{0.755777} = -0.020206$$

$$A = 181° 9' 28''.$$

4. *Azimuth of Polaris without the Ephemeris.*—This method was explained in Art. 88.

***96. Precise Azimuth by Observing Circumpolar Stars.**—The most precise determination of azimuth is made with the theodolite by observing a circumpolar star a number of times in succession. This method yields results correct within 1″

or less. The general method of observing and computing has been explained in Art. 94; here we will consider only a number of additional refinements and corrections.

1. *Schedule of Observations.*—Take two readings on azimuth mark; point to star; read striding level direct and reversed; take four readings on star, recording time and horizontal circle reading; read level; take two more readings on mark. Reverse telescope and repeat, beginning with sighting at the mark.

2. *Curvature Correction.*—In the illustration of Art. 94, the mean of the observed times was used in obtaining the hour angle (t) and from it the azimuth. This mean value of the azimuth requires a correction on account of the fact that a circumpolar star appears to describe an appreciable arc and not a straight line. The U. S. Coast and Geodetic Survey, *Special Publication* 14, gives the following equation for this correction and a table to facilitate its computation.

$$\text{Curvature correction} = -\tan A \cdot \frac{1}{n} \sum \frac{2 \sin^2 \frac{1}{2}\tau}{\sin 1''},$$

in which

n = the number of pointings on the star.

τ = the interval of time between each pointing and the average of the times for the set, expressed in sidereal units.

A = the mean azimuth reckoned from south point clockwise and computed from Eq. (73).

Of course, the negative sign of the correction becomes positive when the star is west of the meridian, as the sign of tan A is negative. The correction is applied to A and is expressed in seconds.

3. *Correction for Aberration.*—On account of the diurnal aberration (Art. 30), a star appears to be displaced from its real position by an amount given in the following equation (U. S. Coast and Geodetic Survey *Special Publication* 14):

$$\text{Correction for diurnal aberration} = -0''.32 \frac{\cos A \cos \phi}{\cos h}.$$

The altitude necessary is either roughly observed or computed from Eq. (8).

4. *Level Correction.*—The inclination, b, of the horizontal axis of the theodolite is obtained by taking readings with the striding level (Art. 66) when the telescope is pointing on a star. Equation

(48) gives the value of b when the striding level is graduated with the zero in the middle, and Eq. (49) when the zero is at one end.

$$\text{Level correction} = b \tan h \qquad [\text{Eq. (50)}].$$

The correction is applied to A and is in the same units as b, usually seconds of arc. If the azimuth mark is not on the same level with the instrument, a similar correction is applied to readings on the mark. Ordinarily the altitude of the mark is too small to affect appreciably the horizontal readings made on it.

5. *Error of Runs.* (Art. 65).—This correction is applied to the mean circle reading on the mark (M), and to the mean circle reading on the star (K).

97. Position of Stars for Most Favorable Determination.—Differentiating Eq. (10), considering A and ϕ variables, we have

$$\cos\phi \cos z \cdot d\phi + \sin\phi \sin z \cos A \cdot d\phi + \cos\phi \sin z \sin A \cdot dA = 0$$

or solving for dA,

$$dA = -\frac{\cos\phi \cos z + \sin\phi \sin z \cos A}{\cos\phi \sin z \sin A} \cdot d\phi,$$

$$= -\frac{\cos\delta \cos t}{\cos\phi \cos\delta \sin t} \cdot d\phi \text{ [making use of Eqs. (13), and (8)].}$$

$$= -\frac{\cot t}{\cos\phi} \cdot d\phi$$

From this we see that an error ($d\phi$) in the assumed latitude will have the least effect on the computed azimuth when the *hour angle* (t) *of the star is close to* 90° *or* 270°.

Similarly differentiating Eq. (8), considering t and A variables, we have

$$\cos t \cos\delta \cdot dt = \sin z \cos A \cdot dA,$$

from which

$$dA = \frac{\cos t \cos\delta}{\sin z \cos A} \cdot dt,$$

$$= \frac{\sin A \cos t}{\sin t \cos A} \cdot dt \text{ [from Eq. (8)],}$$

$$= \tan A \cot t \cdot dt,$$

from which we observe that an error in the hour angle (dt), that is, an error in the assumed time, will have the least effect on the

computed azimuth when the *hour angle* (t) *is* 90° *or* 270° *provided the azimuth* (A) *is nearly* 0° *or* 180°. These two conditions are nearly fulfilled when we *choose circumpolar stars and observe them near elongation.*

The effect of the error in the declination of the star will be insignificant if the declination is taken from the Ephemeris.

98. Circumpolar Stars for Azimuth Determination.—It was shown in the last article that a close circumpolar star not

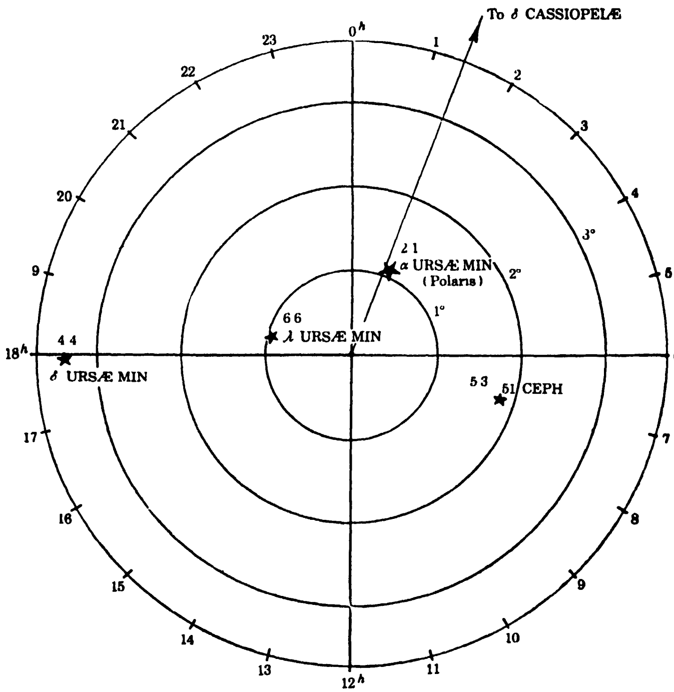


FIG. 56.—Circumpolar stars suitable for azimuth determination.

near the meridian affords a most favorable determination of azimuth. On account of its brightness and nearness to the pole, Polaris is usually chosen. When Polaris is near the meridian, three other stars may be used, δ and λ Ursæ Minoris and 51 Cephei. Figure 56 shows their positions relative to Polaris, their distances from the pole, and their magnitudes. To find these stars in the sky, proceed as follows: (*a*) Orient Fig. 56 relative to meridian at the time of observation, remembering that the pole, Polaris, and δ Cassiopeiæ are in a straight line. (*b*) Having the direction

of the meridian in the figure, estimate the difference in altitude and azimuth between Polaris and the star to be observed. (*c*) Point telescope to Polaris and change its altitude and azimuth by the amounts estimated. This will bring the star sought into the field of the telescope.

99. Azimuth by Observation of the Sun at Any Hour Angle, the Time Being Given.—The method of observing and the process of computing are much the same as those given in Art. 94. Inasmuch as it is impossible to point on the center of the sun, observe the eastern limb, *i.e.*, wait until the limb of the sun *leaves* the *vertical wire* with the point of tangency near the horizontal wire (Figs. 57 and 58). This will give better timing

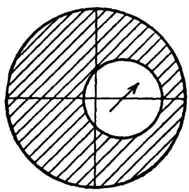


FIG. 57.

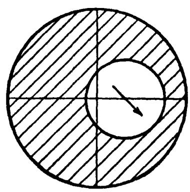


FIG. 58.

FIG. 57.—Observing the eastern limb of the sun in the forenoon, a few seconds before it becomes tangent to the vertical wire, near the horizontal. (Non-inverting telescope.)

FIG. 58.—Observing the eastern limb of the sun in the afternoon, a few seconds before it becomes tangent to the vertical wire near the horizontal. (Non-inverting telescope.)

and will avoid confusing the limbs. No matter in what manner the observation is made, with inverting telescope, prismatic eyepiece, or image projected on paper (Art. 57), *the eastern limb is always observed when the disk of the sun appears to leave the vertical wire.* This will cause the *correction for semidiameter*, s (Art. 51), always to *be added* to the horizontal angle reading on the sun K, provided the angles are measured in a clockwise direction. The altitude of the sun may be observed, or computed from Eq. (8) or (9); it is required for the semidiameter correction. If civil time is used, change it into apparent by the equation of time and obtain the hour angle t ($T_a = 12^h + t$). Either Eq. (71) or (73) may be used for computing A.

SCHEDULE FOR OBSERVING

1. Set instrument over station and level carefully.
2. Point to mark and record circle reading.
3. Release upper plate and point to the sun. Read circle (clockwise), and time when sun leaves the vertical wire near the horizontal.

4. Repeat 3.
5. Record approximate altitude.
6. Point to mark and record circle reading.
7. Reverse telescope and repeat process.

If the instrument lacks a complete vertical circle it will be well to make a morning and an afternoon observation and average the results. The time most favorable for accuracy is when the sun is more than two or three hours from the meridian, preferably near the prime vertical. To illustrate the method of observing and computing, the following determination will be treated in full.

Station: No. 3	*Date:* June 4, 1929	Reading of	Before	After
Lat.: 41° 32′ 13″ N *Long.:* 5^h 26^m $16\overset{s}{.}4$ W	*Limb observed:* Eastern	Local civil clock: Watch:	h m s 15 55 00 16 21 02	h m s 16 16 00 16 42 02

Sight	Watch time	Vernier *A*	Vernier *B*	Vertical angle
		Telescope Direct		
	h m s	° ′ ″	° ′ ″	° ′
Mark......	....	0 0 0	0 0 0	
Sun.......	16 25 26	327 13 20	327 13 20	36 40
Sun.......	16 27 02	327 29 40	327 29 40	
Mark......	....	0 0 0	0 0 0	
		Telescope Reversed		
Mark......	....	0 0 0	0 0 0	
Sun.......	16 31 00	328 9 20	328 9 20	
Sun.......	16 33 21	328 32 20	328 32 20	34 15
Mark.....	... .	0 0 0	0 0 0	
	Watch correction = -26^m 02^s Known civil clock correction = $-6\overset{s}{.}0$.			

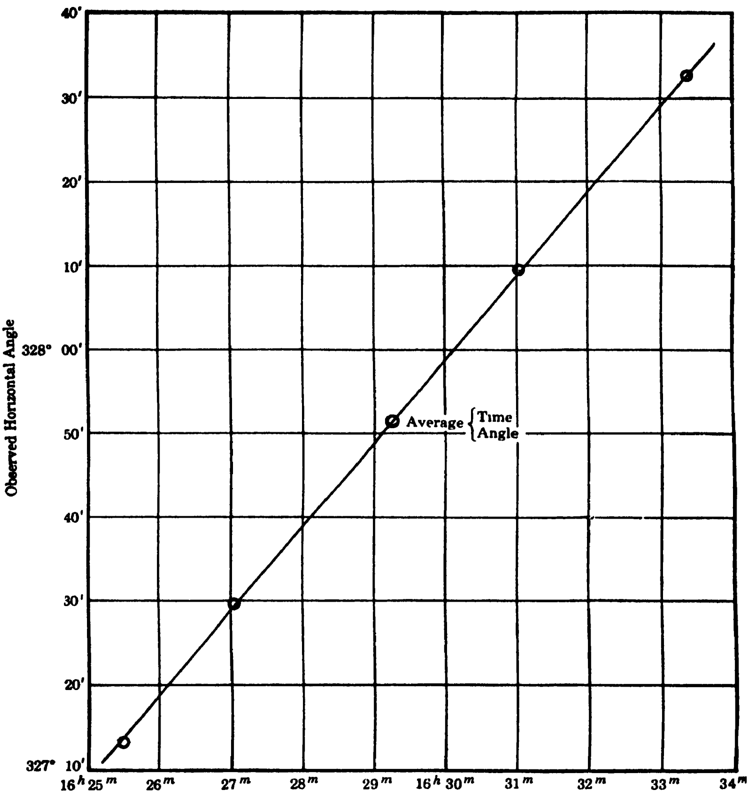


FIG. 59.—Showing that the four observations are consistent.

Reduction

	h	m	s	Reference
Figure 59				Art. 69
Average watch time	16	29	12 3	
Correction, watch to clock		−26	02.0	
Average clock time	16	3	10 3	
Clock correction			−6.0	
Local civil time (T)	16	3	4 3	
Equation of time (E)		+1	52.9	Art. 34
Apparent time of observation (T_a)	16	4	57.2	$T_a - T = E$
Hour angle of sun (t)	4	4	57.2	$T_a = 12^h + t$
	°	′	″	
t in arc	61	14	18	Table I
Latitude of station (ϕ)	+41	32	13	
Declination of sun (δ)	+22	27	32	Art. 34

$$\text{Tan } A = \frac{\sin t \cot \delta \sec \phi}{\tan \phi \cot \delta \cos t - 1}$$

$$a = \tan \phi \cot \delta \cos t$$

Eq. (71) and Eq. (73)

$$\begin{aligned} \log \sin t &= 9\ 942816 \\ \log \cot \delta &= 0.383658 \\ \log \sec \phi &= 0\ 125792 \\ &\overline{0.452266} \end{aligned}$$

$$\begin{aligned} \log \tan \phi &= 9.947373 \\ \log \cot \delta &= 0\ 383658 \\ \log \cos t &= 9\ 682296 \\ \log a &= 0.013327 \\ a &= 1.03116 \end{aligned}$$

$$\begin{aligned} \log (1 - a) &= 8.493597 \\ \log \tan A &= 1.958669 \end{aligned}$$

	°	′	″	Reference
A	89	22	12	
S = semidiameter of the sun from Ephemeris		15	48	
sin z(z = 54° 33″) = 0.8146.				
Correction for horizontal angle reading of sun (s)		19	24	Eq. (42)
Average of horizontal readings on the sun	327	51	10	
Corrected horizontal angle (K)	328	10	34	Corrected for semidiameter
Average of horizontal angle readings on the mark (M)	0	0	0	
$K - M$	328	10	34	
Azimuth of line (A_m)	121	11	38	$A_m = A - (K-M)$ See Fig. 60

100. Azimuth by Observing the Altitude of the Sun When the Time Is Not Known.—The azimuth (A) of a heavenly body as given by Eq. (10) is a function of the latitude (ϕ) of the station, the zenith distance (z) and the declination (δ) of the body in question. Therefore, if the *altitude of the sun or a star is observed at the instant its horizontal angle reading* (K) *is observed*, the azimuth of a line (A_m) can be computed by Eq. (64), as before.

Place the image of the sun in the quadrant in which it *will appear to be leaving both the vertical and horizontal wires.* This will

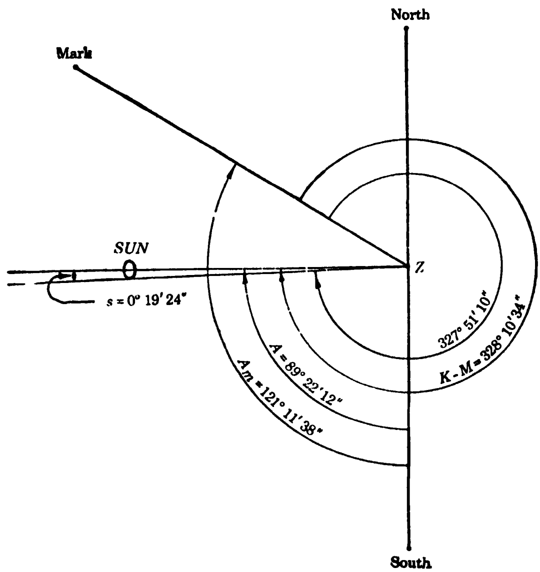


Fig. 60.

always secure good results and give certainty as to the limb observed regardless of the manner of observing. In the forenoon, the eastern and lower limbs are observed; and in the afternoon, the eastern and the upper (Figs. 61 and 62). To avoid altogether the correction for semidiameter, the sun may be placed tangent to both wires in one quadrant, and the horizontal and vertical angles read; and then the image placed in the diametrically opposite quadrant, and the angles read. The mean of the horizontal readings and the mean of the vertical readings will yield the horizontal and vertical readings for the center of the sun.

From the observed altitude obtain the zenith distance (z') and correct it for semidiameter (S) if necessary, refraction (r) and parallax (p). This gives the true zenith distance (z). From the approximate time of observation the declination (δ) of the sun is obtained from the Ephemeris. The latitude (ϕ) is known. The azimuth of the sun may be computed from Eq. (10), *i.e.*,

$$\cos A = \frac{\sin \phi \cos z - \sin \delta}{\cos \phi \sin z}, \tag{75}$$

a simple formula for natural functions and a calculating machine. The data of the determination establish the quadrant; if the sun

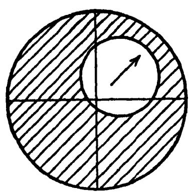


FIG. 61.

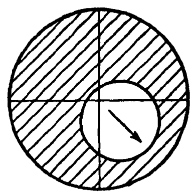


FIG. 62.

FIG. 61.—Observing the eastern and lower limbs of the sun in the forenoon, a few seconds before they become tangent to the vertical and horizontal wires. (Non-inverting telescope.)

FIG. 62.—Observing the eastern and upper limbs of the sun in the afternoon, a few seconds before they become tangent to the vertical and horizontal wires. (Non-inverting telescope.)

is observed west of the meridian, A is less than 180°; if east, A is greater. For a logarithmic solution use Eq. (7) which, when the parts of the astronomical triangle are substituted, becomes:

$$\cot \tfrac{1}{2}A = \pm \sqrt{\frac{\cos \frac{1}{2}[z + (\phi + \delta)] \sin \frac{1}{2}[z + (\phi - \delta)]}{\cos \frac{1}{2}[z - (\phi + \delta)] \sin \frac{1}{2}[z - (\phi - \delta)]}} \tag{76}$$

The data of the problem establish the quadrant of $\frac{1}{2}A$; that is, if the sun is observed west of the meridian, $\frac{1}{2}A$ is in the first quadrant; if east, it is in the second.

SCHEDULE OF OBSERVATIONS

1. Point to mark and read horizontal circle.
2. Release upper plate, point to sun, and read horizontal and vertical circles.
3. Same as 2.
4. Same as 1.
5. Record approximate time.

6. Reverse telescope and repeat process.
7. Record approximate time.

The average of the readings on the mark gives M of Eq. (64); the average of horizontal readings on the sun corrected if necessary for semidiameter [Eq. (42)] gives K. The value of A is computed from Eq. (75) or (76). Then the required azimuth of the mark is, as before,

$$A_m = A - (K - M).$$

If the time is accurately recorded, the correction of the clock may also be obtained by the method of Art. 80 or by first computing the hour angle of the sun (t) by Eq. (74); from which the apparent time is obtained ($T_a = 12^h + t$), and hence the local civil time ($T_a - T = E$).

Example:

Station: No. 1	*Date:* June 20, 1929	Reading of	Before	After
Lat.: 41° 32′ 13″ N	*Observed limbs:* eastern and lower	Watch:	h m s 8 45 23	h m s 9 57 25
Long.: 5^h 26^m 16^s W	*Transit No.* 3	Local civil clock:	8 18 00	9 30 00

Sight	Watch time	Vertical angle	Horizontal angle: Vernier *A*	Horizontal angle: Vernier *B*
		Telescope Direct		
	h m s	° ′ ″	° ′ ″	° ′ ″
Mark...	....	....	0 18 20	0 18 20
Sun....	9 39 37	50 17 00	293 45 20	293 45 20
Sun....	9 42 14	50 44 30	294 19 40	294 19 40
Mark...	....	....	0 18 20	0 18 20
		Telescope Reversed		
Mark...	....	....	0 18 20	0 18 20
Sun....	9 50 55	52 19 00	296 18 20	296 18 20
Sun....	9 53 23	52 45 30	296 52 40	296 52 40
Mark...	....	....	0 18 20	0 18 20

Watch correction = -27^m 24^s.

REDUCTION

	°	′	″	Reference
Average altitude (h')	51	31	30	
Average zenith distance (z')	38	28	30	
Refraction (r)			46	Table IV
Parallax (p)			06	Art. 50
Semidiameter (S)		15	46	A. E.
Corrected zenith distance (z)	38	13	24	$z = z' + r - p - S$
Declination of the Sun (δ)	23	26	40	Art. 34
Given latitude (ϕ)	41	32	13	
$\cos A = \frac{\sin \phi \cos z - \sin \delta}{\cos \phi \sin z}$.				Eq. (75)
$\cos A = 0.265746$; A	285	24	41.	Observation made east of meridian
Average horizontal angle reading on the sun	295	19	00	
$s = \frac{S}{\sin z}$		25	29	Eq. (42)
K	295	44	29	Average $+s$
Average reading on mark, M		18	20	
$K - M$	295	26	09	
Azimuth of the line, A_m	−10	1	28	$A_m = A - (K - M)$
or	349	58	32	

101. Azimuth by Observing the Altitude of the Sun When the Time Is Known.—Equation (74)

$$\sin A = \frac{\sin t \cos \delta}{\sin z}$$

suggests a simple method of computing the azimuth of the sun when the time and the altitude are known. From the given standard or civil time obtain the apparent time (Art. 43), and from it the hour angle (t) of the sun ($T_a = 12^h + t$). The rest of the work follows exactly the lines of the previous article.

Example: In the above illustration, the local civil clock was $6^s.3$ fast. From the time and altitude compute the azimuth of the sun.

	h	m	s
Average reading of watch	9	46	32.3
Watch correction		−27	24.0
Reading of the local civil clock	9	19	08.3
Civil clock correction			−6.3
Correct civil time of observation (T)	9	19	02.0
Equation of time (E)		−1	17.4
Apparent time (T_a)	9	17	44.6
Hour angle of the sun (t)	−2	42	15.4
	°	′	″
t in arc	−40	33	51
Declination of sun (δ)	+23	26	40
Corrected zenith distance (z)	38	13	24

From Eq. (74), and the above values, we have

$$\sin A = -0.964257$$
$$A = -74^\circ\ 38'\ 06''$$
$$= 285^\circ\ 21'\ 54''.$$

Observe that the latitude of the station does not enter this computation.

102. Azimuth by Observing a Star near the Prime Vertical.—Any one of the three methods given above for observing and computing the azimuth of a line by means of the sun are applicable in the case of a star. Preferably observations should not be made near the meridian; to increase the accuracy observe two stars, one east of the meridian and the other west. In the first case (Art. 99), *i.e.*, when the horizontal angle and time are observed, place the star so that it will cross the *vertical wire* near the horizontal and call "Time" as it crosses. In the second case (Art. 100), the star is placed at the intersection of the cross-wires. It is not necessary to record the approximate time, as the declination of the star does not change appreciably in one day. In the third case (Art. 101), the time when the star crosses the intersection of the cross-wires must be known. The altitude is corrected for refraction only. To obtain the hour angle (t) of the star, change the corrected observed time (if civil or standard) into sidereal time (θ), get the right ascension (α) of the star from Ephemeris and solve for t from the equation $\theta = \alpha + t$.

Exercises

1. Compute the azimuth of Polaris at western elongation for June 10, 1929, at Cleveland (lat. 41° 32′ 13″ N; long. $5^h\ 26^m\ 16^s$ W); also the local mean time for same.

2. Check result of Exercise 1, by means of the table in the American Ephemeris or in the Ephemeris of Sun and Polaris.

3. Compute the azimuth of Polaris for 1929, June $2^d\ 22^h\ 50^m\ 10^s$, local civil time of Cleveland.

4. Check result of Exercise 3 by means of some table.

5. Determine the azimuth of a line from the following observations on Polaris.

Station: South Wilson	*Date:* June 23, 1930	Reading of	Before	After
			h m s	h m s
Lat.: 41° 32′ 13″ N	*Star:* Polaris.	Watch:	21 9 11	22 3 10
Long.: $5^h\ 26^m\ 16^s$ W	*Transit:* No. 2	Local civil clock:	20 43 00	21 37 00

Sight	Watch time			Vernier A			Vernier B		
				Telescope Direct					
	h	m	s	°	′	″	°	′	″
Mark..	. . ..			0	0	0	0	0	0
Star.....	21	32	43	22	57	00	22	57	00
Star	21	37	01	22	58	30	22	58	30
Mark	....			0	0	0	0	0	0
				Telescope Reversed					
Mark	....			0	0	0	0	0	0
Star.	21	51	53	23	3	30	23	3	30
Star	21	55	43	23	5	00	23	5	00
Mark.	.....			0	0	0	0	0	0

Known civil clock correction = $+11^{s}.4$.

6. Without the use of the Ephemeris, compute the approximate azimuth (Art. 88) of Polaris for 1932, August 10^{d} 22^{h} 10^{m} 30^{s} local civil time of Cleveland.

7. Determine the azimuth of a line from the following observations on the sun:

Station: No. 4	*Date:* June 20, 1930	Reading of	Before	After
			h m s	h m s
Lat.: 41° 32′ 13″ N	*Limb observed:*	Watch:	17 7 47	17 36 45
Long.: 5^{h} 26^{m} 16^{s} W	eastern	Local civil clock:	16 41 00	17 10 00

Sight	Watch time			Vernier A			Vernier B			Vertical angle	
				Telescope Direct							
	h	m	s	°	′	″	°	′	″	°	′
Mark......	....			0	0	0	0	0	0		
Sun.......	17	16	36	331	58	00	331	58	00	28	20
Sun.......	17	19	25	332	23	20	332	23	10		
Mark......	....			0	0	0	0	0	0		
				Telescope Reversed							
Mark.. ...	....			0	0	0	0	0	0		
Sun.......	17	25	56	333	23	20	333	23	20		
Sun.... . .	17	28	44	333	48	20	333	48	20	26	5
Mark. ...	....			0	0	0	0	0	0		

Known civil clock correction = $+11^s.0$.
Equation of time for the time of observation = $-1^m\ 17^s.4$.
Declination of sun at the time of observation = $23°\ 26'\ 42''$.
Sun's semidiameter = $15'\ 46''$.

8. Compute the azimuth of the sun in Exercise 7, using as the average altitude of its center $27°\ 12'\ 30''$ instead of the given time.

9. Compute the azimuth of the sun in Exercise 7, using the given time and the average altitude of its center ($27°\ 12'\ 30''$).

CHAPTER XI

THE SOLAR ATTACHMENT

An instrument usually mounted upon the telescope of an engineer's transit and used primarily for determining the azimuth of a line by observations on the sun, is called a *solar attachment.* Its particular advantage lies in the fact that practically no computation is involved in determining the azimuth. It is capable of attaining an accuracy of about one minute of arc provided no observations are made when the sun is within one hour of the meridian. Time and latitude may also be determined with the attachment.

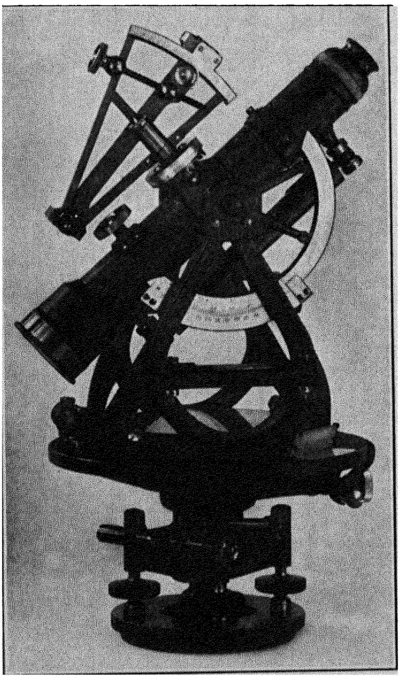


FIG. 63.—The Burt type of solar attachment mounted on engineer's transit. (*W. and L. E. Gurley.*)

103. Description.—Various forms of solar attachments are made but all are about the same in principle and in use. They consist essentially of a *polar axis* perpendicular to the plane formed by the line of sight of the telescope of the transit and the horizontal axis of that telescope; a small telescope, known as the *solar telescope,* which has two motions at right angles to each other corresponding to the two motions of the transit; and either a level attached to the solar telescope or a graduated arc on which the declination of the sun may be set. Figure 63 shows the Burt type of solar attachment having in the place of the telescope a small lens, and in the place of the eyepiece and cross-wires a

metallic surface with lines ruled on it to facilitate the centering of the sun's image. A magnifying glass may be used to observe the reflected image of the sun on this surface. There is also an hour circle graduated to five minutes of time and a declination arc.

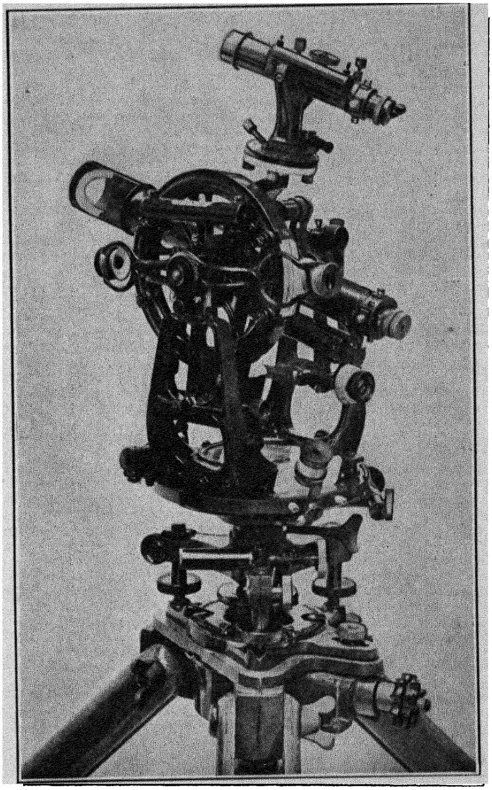


FIG. 64.—The Saegmuller type of solar attachment mounted on engineer's transit. (*C. L. Berger and Sons.*)

Figure 64 shows another common type of attachment. The upper part of the polar axis is Y-shaped and holds the solar telescope. The level, mounted parallel to the solar telescope, is provided with two sights; when the shadow of the one sight appears on the other the solar telescope is directed to the sun. Both the vertical and horizontal motions have clamps and tangent screws. The prismatic eyepiece of the transit, which is provided with the colored glass, fits the eye end of the solar telescope. The line of sight is marked with the usual cross-

wires and on either side of each cross-wire still another wire is placed. This set of wires forms a square with the intersection of the first wires at its center. By setting the image of the sun nearly tangent to all four sides of the square (as in Fig. 65) the center of the sun may be brought into the line of sight.

104. Adjustments.—Before adjusting the solar attachment itself, all the adjustments of the engineer's transit must be carefully checked.

1. *To Make the Polar Axis Perpendicular to the Plane Determined by the Line of Sight of the Transit Telescope and Its Horizontal Axis.*—Carefully level the transit and bring the bubble of the level of the main telescope to the center. Since the horizontal axis and the line of sight of the transit are now parallel with the horizon, the polar axis must be made vertical. Level the solar telescope over one pair of adjusting screws (they are under the plate that holds the polar axis), and turn it 180°. If the bubble is out of center bring it back half way by means of the adjusting screws and the other half by means of the vertical slow motion of the solar attachment. Repeat a number of times over each set of adjusting screws.

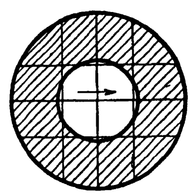
E  W

FIG. 65.—System of wires of solar telescope showing the image of the sun so placed that its center is in the line of sight of the telescope.

2. *To Make the Vertical Cross-wire Truly Vertical.*—This adjustment is exactly the same as in the case of the cross-wires of the main telescope (Art. 55).

3. *To Make the Axis of the Level Parallel to the Line of Sight.*—Make the lines of sight of the two telescopes parallel by pointing both telescopes on the same *distant* object. Level the main telescope and bring the level bubble of the solar telescope to the center of its tube by means of the adjusting screw on the tube.

105. Use of the Solar Attachment.—Suppose that the telescope of the transit is pointed at the intersection of the meridian with the plane of the equator (*i.e.*, the Σ-point, Fig. 66) and that all motions are clamped except that which allows the solar telescope to turn about the polar axis, then the line of sight of the solar telescope will describe a diurnal circle. In particular, if the angle between the two telescopes, when both are in the meridian, equals the declination of the sun at the instant (neglecting, for a moment, refraction), then the line of sight of the *solar*

telescope will follow the sun for a time until the sun's declination changes appreciably. Now suppose that the *exact* direction of the meridian is not known, that the horizontal motion of the transit is unclamped, and that the sun is at least an hour from the meridian; then the only time when the sun may be centered *in the solar telescope is when the transit telescope is in the plane of the meridian.* This is the fundamental principle of the use of the solar attachment.

106. Determination of Azimuth.—The latitude of the place and the approximate time are assumed to be known. From the Ephemeris obtain the declination of the sun (Art. 34) for the

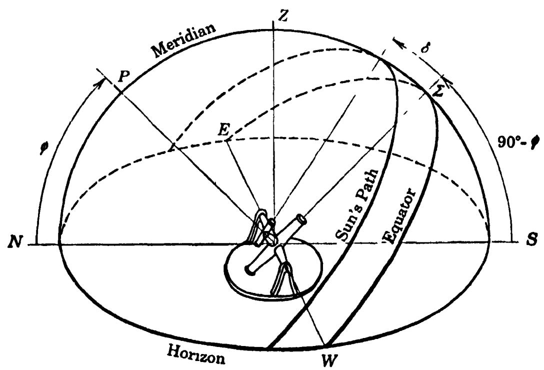


FIG. 66.

time of observation. To allow for refraction, the altitude of the sun is roughly observed (or computed from the approximate time) and the amount of the refraction is obtained from Table IV and added algebraically to the declination. The correction may also be obtained from handbooks published by a number of instrument makers, in which it is given for different latitudes and hour angles of the sun. Having the latitude of the place and the declination of the sun, we are ready to begin observation.

The transit is set up at the station and the two telescopes are pointed on a distant mark to bring them into the same vertical plane. *Clamp the horizontal motion of the solar telescope.* Set the vertical circle of the transit to read the corrected declination of the sun (for δ negative, objective end of transit telescope up; for δ positive, down); then level the solar telescope and *clamp its vertical motion.* This completes the work of making the

vertical angle between the two telescopes equal to the corrected declination of the sun.

Set the two plates of the transit at 0° and by use of the slow motion of the lower plate point the transit on the mark, the azimuth of which is required. Raise the transit telescope to the co-latitude of the place and clamp *this vertical motion.*

We are now ready to observe the sun. The transit telescope *is turned to the approximate known direction of the meridian.* The vertical motions of the two telescopes are kept clamped, as well as the lower plate of the transit, while the upper plate of the transit and the horizontal motion of the solar telescope are left free. By means of these two free motions observe the sun *through the solar telescope.* The disk of the sun is placed in the manner shown in Fig. 65 and the sun will move parallel to one set of wires, as shown, when the *transit telescope is on the meridian.* The reading of the horizontal circle gives at once the azimuth of the line.

107. Determination of Time.—When the transit telescope is in the meridian and the sun's center in the line of sight of the solar telescope, the *angle* between the two telescopes measured in a plane perpendicular to the polar axis is evidently equal to the hour angle of the sun. To measure this angle, point the transit telescope to a distant mark nearly on the same level as the instrument and read the horizontal circle. *Carefully level this telescope,* point the solar telescope to the same mark and again read the horizontal circle. As a rule this will necessitate slightly raising or lowering the line of sight of the solar telescope. The difference between the two readings gives the hour angle of the sun; hence, by comparing the clock time of observation with the time computed from the hour angle of the sun, the correction of the clock may be obtained.

Exercise

Describe the method of determining the latitude of the place and the clock correction by meridian observations with the solar attachment.

CHAPTER XII

DETERMINATION OF TIME BY THE TRANSIT INSTRUMENT

108. The transit instrument is designed primarily to be used for the accurate determination of time. It consists essentially of a telescope, a horizontal axis perpendicular to the telescope, and a reticle at the focal plane of the objective. The horizontal axis, or the axis of rotation, rests on Y's and is mounted in an east-and-west direction which makes the telescope move in the meridian. A graduated circle is attached to the instrument in order to set the proper zenith distance. The reticle (Fig. 69) consists of an odd number of vertical threads, or wires, generally placed symmetrically with respect to the middle wire, and two horizontal wires. Suppose there are five vertical wires, and let t_1, t_2, t_3, t_4, t_5 represent the times of transit of a star over these wires; then the average of the t's will give the time θ_m that the star will cross a fictitious wire called the *mean wire, i.e.*,

$$\theta_m = \frac{t_1 + t_2 + t_3 + t_4 + t_5}{5} \tag{77}$$

The instrument is said to be in perfect adjustment when the mean wire traces the meridian, as the telescope rotates on its axis. If the telescope is in perfect adjustment we have for the correction of the clock,

$$\Delta\theta = \alpha - \theta_m \tag{78}$$

To adjust the instrument and keep it in perfect adjustment is not possible; therefore, the time θ_m must be corrected for three principal instrumental errors. For the sake of simplicity we assume that the east support of the axis of rotation is fixed.

1. *Azimuth constant a* is the horizontal angle that the axis of rotation makes with the true east and west line. *It is assumed to be positive when the west end of the axis is toward the south.*

2. *Level constant b* is the angle that the axis of rotation makes with the plane of the horizon. *It is assumed to be positive when the west end is high.*

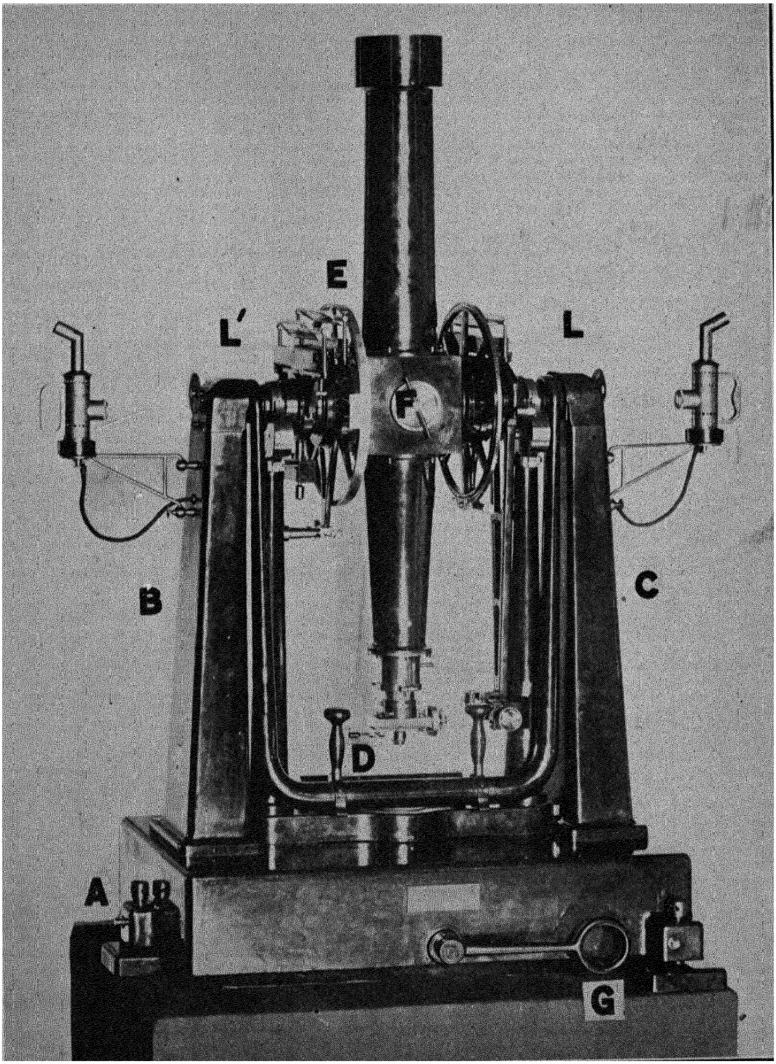


FIG. 67.—Four-inch transit instrument. There are three foot-screws supporting the instrument, one at A and two on the other side. The axis of rotation LL' rests on the supports B and C. The frame of the level D is U-shaped and rests on the axis of rotation at L and L'. The circle E is to set the zenith distance. The light from the lamps passes through the hollow axis of rotation and is reflected by means of a mirror within F to illuminate the field. The instrument is reversed by means of the arm G. (*The Warner and Swasey Co.*)

The line through the optical center of the objective, and perpendicular to the axis of rotation, is called the *collimation axis.* The line joining the optical center of the objective with the mean wire is called the *line of sight.*

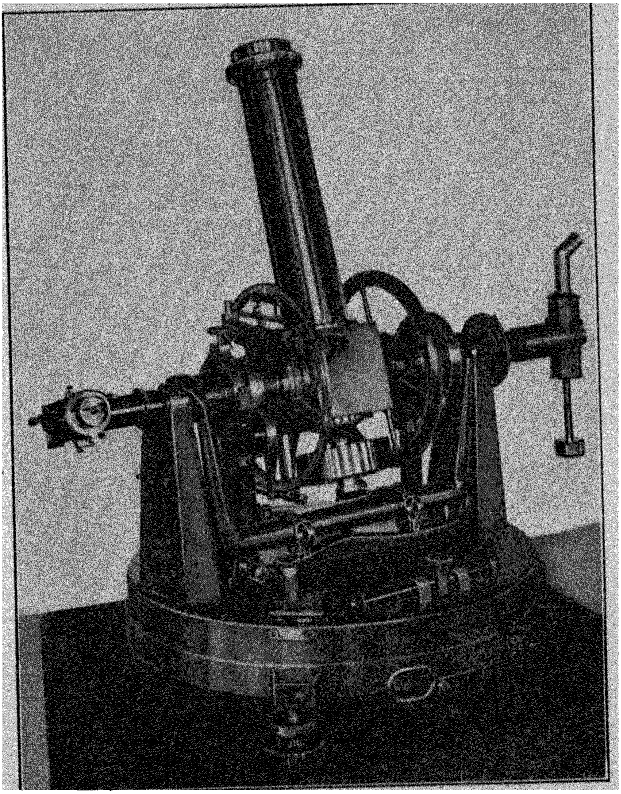


FIG. 68.—Broken-telescope transit. A prism in the optical axis of the telescope directs the rays of light at right angles through the hollow horizontal axis of the instrument to the eye end. With this arrangement it is possible to make all observations from one position. In all other respects this instrument is the same as the one shown in Fig. 67. (*The Warner and Swasey Co.*)

3. *Collimation constant c* is the angle formed by the collimation axis and the line of sight. *It is positive when the mean wire is west of the collimation axis.*

From the above it may be seen that when we are observing a star south of the zenith and when a is positive, the star will cross the mean wire too early. Likewise, for any star above the pole, when b or c is positive, the star will be observed too early and hence the correction to θ_m should be positive.

109. Theory of Instrumental Errors.—From Figs. 71 and 72 and the above definitions of a, b, and c, we may write

$$PZ = 90^\circ - \phi. \qquad PR = 90^\circ - \delta.$$
$$\angle HZM = 90^\circ - a. \qquad HZ = 90^\circ - b.$$

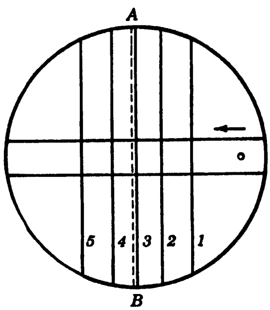


Fig. 69.

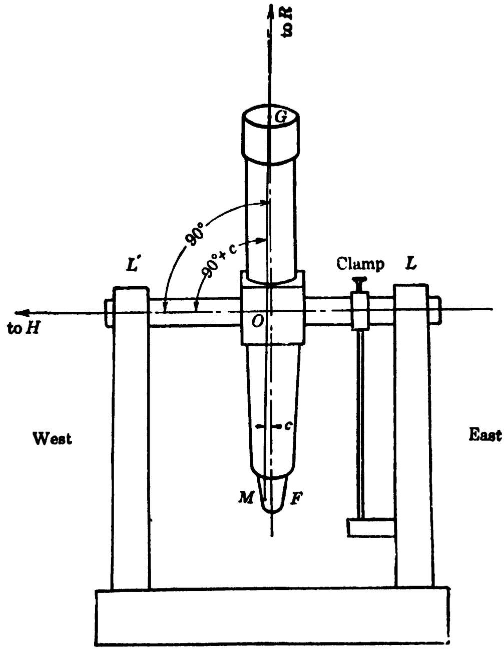


Fig. 70.

Fig. 69.—Reticle of the transit instrument. When the star appears in the field it is made to move between the two horizontal wires by slightly changing the zenith distance. The time is noted as the star crosses each of the five vertical wires. The average of the time of these transits gives the time that the star will cross the fictitious wire AB (mean wire).

Fig. 70.—The line GF through the optical center of objective G and perpendicular to the axis of rotation LL' is the axis of collimation. The line GM through G and the mean wire M is the line of sight. The collimation constant, the angle c, is shown positive (see page 161). If the axis LL' is turned end to end M will be east of F and hence c negative.

Figure 70 shows that the angle HOR is $90^\circ + c$. This angle is measured by the arc HR of Figs. 71 and 72. Hence,

$$HR = 90^\circ + c.$$

Let t be the hour angle of R, *measured toward the east and in seconds of time.* For convenience let $ZM = n$, $MR = f$, $\angle PMR = m$. From the law of sines in the spherical triangle PMR, we have

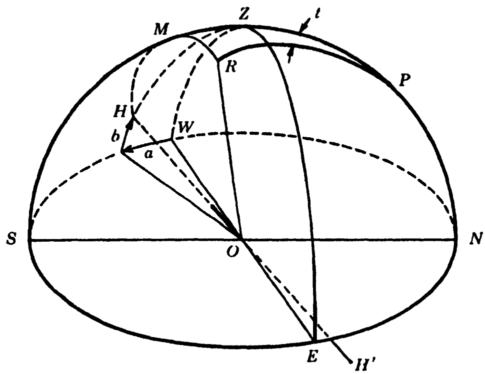


FIG. 71.—SZN represents the meridian, P the celestial pole, WE the east and west line, H and H' the points where the axis of rotation produced pierces the celestial sphere, and R the position of a star as it crosses the mean wire of the telescope.

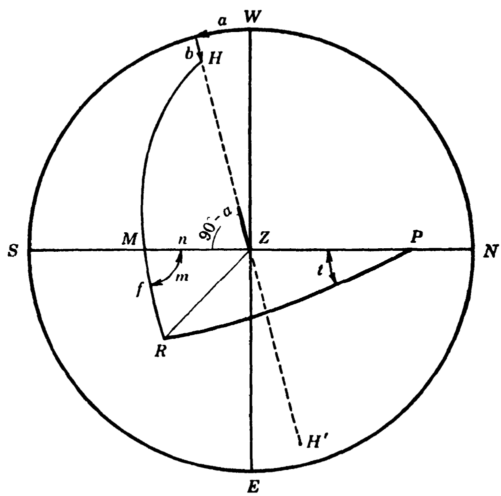


FIG. 72.—The projection of Fig. 71 on the plane of the horizon. PR is the hour circle of R and HR an arc of great circle connecting H and R.

$$\frac{\sin t}{\sin f} = \frac{\sin m}{\sin (90° - \delta)}$$

or

$$\sin m \sin f = \sin t \cos \delta \tag{79}$$

and from the law of cosines in the spherical triangle HMZ, we have

$$\cos HM = \cos (90° - b) \cos n + \sin (90° - b) \sin n \cos \angle HZM$$

or

$$\cos (90° + c - f) = \sin (f - c) = \sin b \cos n + \cos b \sin n \sin a. \quad (80)$$

Since a, b, and c are very small (they are usually less than 1^s), we may write in Eq. (79)

t in the place of $\sin t$,
f in the place of $\sin f$,
1 in the place of $\sin m$,

that is,

$$f = t \cos \delta. \quad (81)$$

Likewise, in Eq. (80) we may write

$f - c$ in the place of $\sin (f - c)$,
b in the place of $\sin b$,
1 in the place of $\cos b$,
a in the place of $\sin a$,
$\phi - \delta$ in the place of n,

so that Eq. (80) takes the form,

$$f - c = b \cos (\phi - \delta) + a \sin (\phi - \delta). \quad (82)$$

Subtracting Eq. (81) from Eq. (82) and solving for t, we have

$$t = a \sin (\phi - \delta) \sec \delta + b \cos (\phi - \delta) \sec \delta + c \sec \delta.$$

It is evident that a, b, and c are here expressed in seconds of time, inasmuch as t was assumed to be expressed in those units.

To abbreviate the coefficients of a, b, and c, let

$$A = \sin (\phi - \delta) \sec \delta. \quad (83)$$

$$B = \cos (\phi - \delta) \sec \delta. \quad (84)$$

$$C = \sec \delta. \quad (85)$$

Therefore,

$$t = aA + bB + cC. \quad (86)$$

Inasmuch as t is the hour angle of the star R, and θ_m represents the observed time at which the star crossed the mean wire, the time that it will cross the meridian will be $\theta_m + t$. Therefore, the correction of the clock is

$$\Delta\theta = \alpha - (\theta_m + t). \quad (87)$$

110. Clock Correction.—To determine the clock correction $\Delta\theta$, a number of stars are observed and the average $\Delta\theta$ obtained. This, however, does not take account of the rate of the clock. *The correction due to rate may be* obtained as follows: let

θ_1 be the time of transit of the first star, in hours and fractions of hours.

θ_n be the time of transit of the nth star, in hours and fractions of hours.

$d\theta$ be the rate of the clock per hour in seconds; positive when the clock is losing, negative when the clock is gaining.

Therefore, the clock time of each star should be increased by

$$(\theta_n - \theta_1) \cdot d\theta.$$

It has been indicated in Art. 30 that the time of transit of a star should be corrected for the effect of aberration. Equation (27) gives for this correction $-0^s.021 \sec \delta \cos \phi$, which, by Eq. (85), becomes $-0^s.021C \cos \phi$.

The final form of the correction of the clock $\Delta\theta$ as given in Eq. (87) will be

$$\Delta\theta = \alpha - [\theta_m \overbrace{- 0^s.021C \cos \phi}^{\text{aberration}} + \overbrace{(\theta_n - \theta_1)d\theta}^{\text{clock rate}} + \overbrace{Bb + Cc + Aa}^{t}]. \quad (88)$$

Equations (83), (84), and (85) are known as Mayer's formulas and in the form in which they are given refer to stars at upper transit. *For stars at lower transit* substitute for δ, $180° - \delta$. They become

$$A_n = \sin(\delta + \phi) \sec \delta, \; B_n = \cos(\delta + \phi) \sec \delta, \; C_n = -\sec \delta. \quad (89)$$

A, B, and C are functions of ϕ and δ and for a fixed observatory tables[1] are made with δ as the argument.

111. Determination of Instrumental Constants. *Level Constant b.*—The level is supported at the ends of the axis of rotation as is shown in Fig. 67. It has been assumed that the east end of the axis of rotation is fixed; hence, in the case where the zero of the graduations of level is at one end of the tube, the level is

[1] A table for A, B, and C is given in *Special Publication* 14, U. S. Coast and Geodetic Survey.

considered direct when the zero is east. This has been explained in Art. 66. Readings *must be taken with level direct and reversed,* to eliminate the error of adjustment of level in its frame.

The value of b may be computed from Eq. (48) or (49). This value, however, cannot be assumed to remain the same during a long interval of time (say one hour). To obtain the value of b at the time a given star is observed, we record the approximate sidereal time when level readings are taken, and interpolate for the time required.

Collimation Constant c.—Neglecting the rate of the clock, the terms of Eq. (88) may be grouped for convenience as follows:

$$\Delta\theta = \alpha - [\overbrace{\theta_m - 0.021C \cos\phi + Bb}^{\theta_b} + Cc + Aa] \qquad (90)$$

or

$$\Delta\theta = \alpha - [\theta_b + Cc + Aa]. \qquad (91)$$

Equation (83) shows that the value of A approaches zero when the zenith distance of a star approaches zero, so that Eq. (91) *for a star near the zenith becomes* approximately

$$\Delta\theta = \alpha - [\theta_b + Cc]. \qquad (92)$$

If the telescope is reversed and another star is observed the mean wire will move to the other side of the collimation axis (see Fig. 70). This will change the sign of the collimation correction Cc. Hence, to keep track of the proper sign of this correction, we must keep track of the reversals of the instrument. For this reason we make an arbitrary assumption as follows: *When the clamp of the telescope is east, c will be assumed positive when the mean wire is west of the collimation axis.*

That is, we have

$$\Delta\theta = \alpha' - [\theta'_b + C'c]$$

for a zenith star with clamp east. When the instrument is reversed, we must have,

$$\Delta\theta = \alpha'' - [\theta''_b + C''(-c)]$$

for a zenith star with clamp west. These last two equations have two unknowns, namely, $\Delta\theta$ and c (for clamp east). Eliminating $\Delta\theta$ and solving for c, we have

$$c = \frac{(\alpha' - \theta'_b) - (\alpha'' - \theta''_b)}{C' + C''} \qquad (93)$$

This gives the value of c for clamp east. To obtain the value for clamp west take the negative of this result.

Azimuth Constant a.—Neglecting the rate of the clock, and regrouping all the terms now known, of Eq. (88), we have

$$\Delta\theta = \alpha - [\overbrace{\theta_m - 0^s.021C\cos\phi + Bb + Cc}^{\theta_c} + Aa] \tag{94}$$

or

$$\Delta\theta = \alpha - [\theta_c + Aa]. \tag{95}$$

A similar equation for a second star will be

$$\Delta\theta = \alpha' - (\theta_c' + A'a). \tag{96}$$

Subtracting Eq. (95) from Eq. (96) and solving for a, we get

$$a = \frac{(\alpha - \theta_c) - (\alpha' - \theta_c')}{A - A'}. \tag{97}$$

It is evident that to make the error of the determination of a as small as possible, the denominator of Eq. (97) must be numerically as large as possible. This is accomplished by choosing stars for which A and A' have large values opposite in sign. A study of Eq. (83) shows that we must choose *one star near the pole and the other as far south as possible*, or *two circumpolar stars, one at upper and the other at lower culmination.*

112. Adjustments of the Transit Instrument.—In the above discussion it has been assumed that the instrument has been adjusted and that the values of a, b, and c were made as small as possible. Here we shall consider these adjustments.

1. *To Make the Axis of Rotation Horizontal.*—Take level readings both direct and reversed and obtain the value of b. By means of the leveling screws raise or lower the west end of the axis in accordance with the sign of b. Repeat the process until b is nearly zero.

2. *To Make the Line of Sight Perpendicular to the Axis of Rotation.*—Point the telescope toward a well-defined mark and bisect it with the middle wire of the reticle. Reverse instrument and point the telescope upon the same mark. If the middle wire still bisects the mark, no adjustment is necessary; if not, move the wire half way toward the mark by means of the slow-motion screws that adjust the azimuth. Then bisect the mark with the middle wire by moving the whole set of wires, using the

screws on the eyepiece tube. Reverse the instrument and repeat the process until the adjustment is made.

3. *To Make the Axis of Rotation Point in the East and West Direction.*—Let us suppose the instrument set so that the telescope swings approximately on the meridian. To obtain the approximate correction of the clock observe the time of transit of a star within 10° of the zenith. Having already made the values of b and c very small, the difference between the R.A. of the star and time of its transit will give a good approximation to the clock correction. Now observe a slow-moving star (declination about 75° or more). When the star is near the middle wire bisect it by means of the azimuth slow-motion screws. Follow the bisection by means of the same screws until the clock indicates that the star is on the meridian. Repeat with another star.

113. Observing List.—The requirements of a good set of stars for determining time can be outlined as follows:

a. The same number of stars north and south of the zenith, for the error in azimuth will then be practically eliminated.

b. Two zenith stars, one clamp east and the other clamp west, thus eliminating the azimuth correction as far as possible from the determination of c.

c. One star near and above the pole and the other as far south as possible or below the pole. This will tend to make the denominator of Eq. (97) as large numerically as possible.

The observing list has the following headings:

No.	Star	Magn.	R.A.	Declination	Setting	N or S

Columns 2, 3, 4, and 5 are obtained from the Ephemeris. The magnitude serves to identify stars. The right ascension can be taken to the nearest second and the declination to the nearest minute.

The sixth column is obtained from

$$z = \phi - \delta \text{ for south of zenith stars,}$$
$$z = \delta - \phi \text{ for north of zenith stars, and above pole.}$$

The last column designates whether the star is south or north of the zenith.

114. Directions for Observation.—It is clear that the object of this determination is to find the correction $\Delta\theta$ of the clock. Hence a clock is indispensable. The seconds of the clock are automatically recorded by means of an electric circuit on an instrument known as a chronograph (Fig. 73). Usually the beginning of each minute is indicated on the sheet or tape of the chronograph by a mechanism in the clock. Before the work of observing stars begins, the hour and minute corresponding to this particular mark are noted on the sheet of the chronograph.

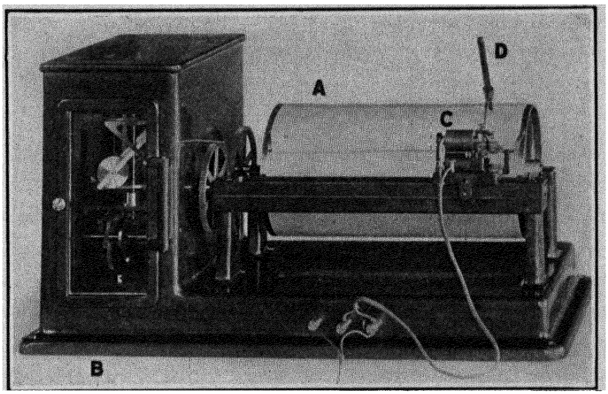


FIG. 73.—A Chronograph. The cylinder A rotates, by means of a clock arrangement B, once a minute. The beat of the clock is transmitted by an electric circuit to the electromagnet C which releases the pen D for a small fraction of a second, thus marking the seconds of the clock on the sheet of paper wrapped around the cylinder. (*The Warner and Swasey Co.*)

Another electric connection from the chronograph leads to a key at the telescope. By pressing the key the observer interpolates signals among the clock markings on the sheet. Thus the exact time at which the star crosses each wire of the reticle can be estimated to the nearest .01 of a second.

After reading the level direct and reversed, the circle for reading zenith distance is set for the first star. The clamp of the axis of rotation had better not be used, as there is a slight tendency to raise one end of the axis. When the star appears in the field it can be brought between the horizontal wires of the reticle (see Fig. 69) by gently tapping the telescope. Center the eyepiece opposite the five wires, and press the key when the star passes over each vertical wire.

SCHEDULE OF OBSERVATIONS

1. Have clamp of telescope east and level with zero east.
2. Take level readings and record approximate sidereal time. (Care should be taken to give time for level bubble to come to rest before reading.)
3. Observe half the stars, at least one star to be within 8° of the zenith.
4. Take level readings as before.
5. Reverse telescope.
6. Take level readings.
7. Observe the remaining stars, at least one star to be within 8° of the zenith.
8. Take level readings.

115. Example for Reducing Observations.—Page 171 gives a set of eight stars for the determination of the clock correction. Their declinations are given to the nearest minute and are to be used for obtaining the corresponding values of A, B, and C.

The apparent right ascensions are given in line marked "α" and they are computed correct, as far as possible to the nearest five-thousandth of a second.

The chronograph readings (t_1, t_2, . . . ,) for the time of transit of stars over the five wires of the reticle are read to the nearest hundredth of a second and are recorded on the lines marked "Wire transits."

The values of A, B, and C are obtained (correct to three decimals) from the observatory book of star constants.

θ_m is the mean of the t's and is computed to the nearest five-thousandth of a second.

The aberration is computed (to nearest five-thousandth) from equation $K' = -0^s.021\ C \cos \phi$.

The rate of the clock was too small to be included in this determination.

The Value of b.—The four values of b are computed from Eq. (49), $d = 0^s.057$. The value of b for each star is obtained by interpolating, using θ_m for time.

The Value of c.—The value of c is determined from Eq. (93). Choose one clamp-east star nearest to zenith, in this case ν^1 Bootis, which gives

$$\alpha' = 15^h\ 28^m\ 24^s.800.$$
$$\theta_b' = 15^h\ 27^m\ 43^s.455\ [\theta_b \text{ defined in Eq. (90)}].$$
$$C' = 1.326.$$

WARNER AND SWASEY OBSERVATORY—C.S.A.S.—COURSE IN PRACTICAL ASTRONOMY

DETERMINATION OF TIME WITH TRANSIT INSTRUMENT

Transit: 3-in. *Observer:* Van Voorhis-Hogan.

Clock: Reifler No. 56. *Computer:* Van Voorhis-Hogan. *Date:* June 6, 1929.

$$\Delta\theta = \alpha - (\theta_m - 0^s.021\ C \cos\phi + Bb + Cc + Aa)$$

Star	δ Bootis	γ Ursæ Minoris	ι Draconis	ν^1 Bootis	α Serpentis	β^1 Scorpii	ϕ Herculis	δ Ophiuchi
Declination	+33° 35′	+72° 05′	+59° 13′	+41° 04′	+6° 39′	−19° 37′	+45° 07′	−3° 31′
Clamp	East	E	E	E	West	W	W	W
Wire transits	56 58	04 47	37 82	40 80	04 00	36 05	49 29	55 19
	58 37	10 15	(41 05)	42 77	05 46	37 78	51 53	56 79
	15h 12m 00 47	15h 20m 15 40	15h 22m 44 39	15h 27m 45 08	15h 40m 07 48	16h 00m 39 29	16h 05m 53 84	16h 09m 58 68
	02 35	20 63	47 70	47 25	09 30	41 27	56 51	00 35
	04 43	(26 31)	50 87	49 52	10 81	42 80	58 53	01 95
B	1 189	2 800	1 862	1 326	0 826	0 512	1 415	0 708
C	1 202	3 251	1 955	1 326	1 007	1 062	1 417	1 001
A	0 166	−1 653	−0 594	0 012	0 576	0 930	−0 088	0 709
θ_m	15 12 00 440	15 20 15 390	15 22 44 365	15 27 45 085	15 40 07 410	16 00 39 440	16 05 53 940	16 09 58 590
Aberration	− 0 020	− 0 050	− 0 030	− 0 020	− 0 015	− 0 015	− 0 020	− 0 015
Bb	− 1 465	− 3 420	− 2 275	− 1 610	− 1 010	− 0 625	− 1 725	− 0 860
Cc	− 0 225	− 0 605	− 0 365	− 0 245	+ 0 185	+ 0 200	+ 0 265	+ 0 185
Aa	− 0 105	+ 1.055	+ 0 380	− 0 010	− 0 365	− 0 590	+ 0 055	− 0 450
θ'	15 11 58 625	15 20 12 370	15 22 42 075	15 27 43 200	15 40 06 205	16 00 38 410	16 05 52 515	16 09 57 450
α	15 12 40 250	15 20 54 085	15 23 23 700	15 28 24 800	15 40 47 845	16 01 20 125	16 06 34 120	16 10 39 110
$\Delta\theta$	41 625	41 715	41 625	41.600	41 640	41 715	41 605	41 660

Level Constant, $b = \frac{1}{4}[W + E - (W' + E')]d \qquad d = 0^s057$

Time	Level Direct W	Level Direct E	Level Reversed W′	Level Reversed E′	Clamp	b
15h 12m	50 0	9 0	52 0	93 4	E	−1 231
15 28	50 8	9 5	52 2	93 3	E	−1 214
15 40	51 6	10 0	53.0	94 5	W	−1 224
16 10	51 6	10 0	52 2	94 9	W	−1 218

Collimation Constant

$$c = \frac{(\alpha' - \theta'_b) + (\alpha'' - \theta''_b)}{C' + C''}.$$

$c = -0^s.186$ Clamp E

$c = +0\ 186$ Clamp W

Azimuth Constant

$$a = \frac{(\alpha - \theta_e) - (\alpha' - \theta'_e)}{A - A'}.$$

$a = -0^s637$

$\Delta\theta$ (mean) $= +41^s648.$

Similarly, the nearest-to-zenith clamp-west star is ϕ Herculis, so that

$$\alpha'' = 16^h\ 06^m\ 34\overset{s}{.}120.$$
$$\theta_b'' = 16^h\ 05^m\ 52\overset{s}{.}195.$$
$$C'' = 1.417.$$

Therefore

$$c = \frac{41\overset{s}{.}345 - 41\overset{s}{.}925}{2.743} = -0\overset{s}{.}211.$$

For clamp west,

$$c = +0\overset{s}{.}211.$$

The Value of a.—To obtain a, Eq. (97) is used. The star nearest to the pole is γ Ursæ Minoris, so that

$$\alpha = 15^h\ 20^m\ 54\overset{s}{.}085.$$
$$\theta_c = 15^h\ 20^m\ 11\overset{s}{.}235\ [\theta_c \text{ defined in Eq. (94)}].$$
$$A = -1.653.$$

The most southern star is β^1 Scorpii, for which

$$\alpha' = 16^h\ 01^m\ 20\overset{s}{.}125.$$
$$\theta_c' = 16^h\ 00^m\ 39\overset{s}{.}025.$$
$$A' = 0.930.$$

So that

$$a = \frac{42\overset{s}{.}850 - 41\overset{s}{.}100}{-2.583} = -0\overset{s}{.}678$$

More Accurate Values of c and a.—In computing the value of c it was assumed that the term Aa for the zenith stars was zero; however, this is not the case. To compute a more accurate value of c, Eq. (91) may be used for the two zenith stars. That is

$$\Delta\theta = 15^h\ 28^m\ 24\overset{s}{.}800 - [15^h\ 27^m\ 43\overset{s}{.}455 + 1.326c + (0.012)\cdot(-0\overset{s}{.}678)].$$

$$\Delta\theta = 16^h\ 06^m\ 34\overset{s}{.}120 - [16^h\ 05^m\ 52\overset{s}{.}195 - 1.417c + (-0.088)\cdot(-0\overset{s}{.}678)]$$

Solving for c, we obtain $c = -0\overset{s}{.}186$ for clamp east.

Having this new value of c a new value of a is computed as before from Eq. (97), which is

$$a = -0\overset{s}{.}637.$$

Cc and Aa.—These two lines are now computed correct to the nearest five-thousandth of a second.

θ'.—Includes all the terms in the bracket of Eq. (90).

$$\Delta\theta = \alpha - \theta'.$$

Observe the following:

1. The sign of b was found to be negative, hence the west end of the axis of rotation was low.

2. The sign of c for clamp east was found to be negative, hence the mean wire was east of the collimation axis when the clamp was east.

3. The sign of a was found to be negative, hence the west end of the axis of rotation was towards the north.

116. Least-squares Solution.—A more accurate reduction of the above observations may be made by *least squares*. Equation (90) involves three unknowns, $\Delta\theta$, c, and a. Since we have as many such equations as stars observed, the values of these unknowns may be obtained by forming their normal equations.

To avoid dealing with large numbers, let $\Delta\theta_0$ be an approximate known value of the clock correction $\Delta\theta$. Let x be the unknown correction to $\Delta\theta_0$, then

$$\Delta\theta = \Delta\theta_0 + x. \tag{98}$$

Hence Eq. (90) takes the form:

$$x + \Delta\theta_0 = \alpha - (\theta_b \pm Cc + Aa)$$

with the plus sign before Cc for clamp-east stars and the minus for clamp-west, or

$$\pm Cc + Aa + x = \overbrace{\alpha - \theta_b - \Delta\theta_0}^{known}.$$

Assuming in the above illustration $\Delta\theta_0 = +41^s000$, the corresponding observation equations are:

$$\begin{aligned}
1.202c + 0.166a + x &= 0.295\\
3.251c - 1.653a + x &= 1.165\\
1\ 955c - 0.594a + x &= 0.640\\
1.326c + 0.012a + x &= 0.345\\
-1.007c + 0.576a + x &= 0.460\\
-1.062c + 0.930a + x &= 0.325\\
-1.417c - 0.088a + x &= 0.925\\
-1.001c + 0.709a + x &= 0.395.
\end{aligned}$$

Assuming unit weight for each of these equations, we have for the normal equations

$$22.746c - 8.473a + 3.247x = 3.336 \text{ normal for } c$$
$$-8.473c + 4.820a + 0.058x = -1.487 \text{ normal for } a$$
$$3.247c + 0.058a + 8.000x = 4.550 \text{ normal for } x.$$

Solving simultaneously, we obtain

$$c = -0^{s}.184;\ a = -0^{s}.639;\ x = 0^{s}.648;$$

and

$$\Delta\theta = +41^{s}.648.$$

The agreement with the previous solution is somewhat unusual.

CHAPTER XIII

LONGITUDE

117. General Considerations.—The astronomical longitude of a place is the angle between an arbitrarily chosen initial meridian plane, and the meridian plane of the place. The meridian through Greenwich Observatory, near London, England, has been almost universally adopted as the zero meridian.

The *determination of the longitude of a place is made by determining the local time at the place and comparing that time with the local time of the same instant at Greenwich or at some other place of known longitude.* This has already been expressed in Art. 27 thus: *The difference between the corresponding local times of two places gives their difference in longitude.* In the United States all local times are compared with the local time of the U. S. Naval Observatory (Washington), the longitude of which is taken as $5^h\ 8^m\ 15^s.784$ W. In 1926 a more accurate determination ($5^h\ 8^m\ 15^s.751$ W) was made by wireless, but for the sake of consistency all longitudes are referred to the older value.

The principal method of determining longitude may be outlined as follows:

1. Determine the local time at two stations, at one of which the longitude is known and the other unknown.
2. Signal the local time of the former station to the latter.
3. The difference of the two local times expresses the difference in longitude of the two stations.

118. Longitude by Meridian Transit of the Moon.—Just as the sun appears to move among the stars and its right ascension and declination change continually, the moon, on account of its revolution about the earth, appears to move among the stars, but much faster than the sun. The right ascension of the moon is given in the American Ephemeris for every hour of Greenwich time. If, at the station whose longitude is to be found, the sidereal time is known (Chap. VIII), observation of the sidereal time of the moon's meridian transit yields its right ascension. From the Ephemeris the Greenwich time corresponding to this

right ascension is obtained. The difference of the two times (both expressed in the same units, sidereal or civil) gives the longitude required. This is, of course, a very rough determination but is still used in remote places.

119. Transportation of Chronometers.—If a number of chronometers keeping the local time of the station whose longitude is known are transported to another station of unknown longitude and compared with the local time of this station, the difference between the two times will give the difference in longitude between the two stations. A number of chronometers are used in both stations simply to increase the accuracy of the determination, and the errors and rates of the chronometers are carefully determined. Before the laying of the first Atlantic cable the longitudes of New York and of other Atlantic ports were determined by this method. Essentially this same method is used in determining the longitude of a ship at sea; one or more chronometers on board ship give Greenwich Civil Time and from astronomical observations the ship's local time is obtained.

120. Telegraphic Method.—The most exact method of determining longitude previous to the introduction of the wireless was by telegraphic signals. A night's observation at each station consists in determining the local time by means of the transit instrument as explained in Chap. XII, using the same list of stars at both stations, if possible. Then, at the time previously agreed upon, the observer at the western station sends, by tapping the telegrapher's key, 15 or 20 signals which are recorded at the chronographs of both stations; after this the observer at the eastern station sends, in the same manner, about twice as many signals which are recorded at both chronographs; and then the western observer sends 15 or more signals. Finally, another set of stars is observed at each station for a second time determination. Each chronograph records also the second-beats of the local chronometer. The error of the chronometer is computed from the transit observation, and thus the *true local time* of each arbitrary signal of the exchange may be ascertained. This is done at each station and the results sent to the other. The difference of the two local times for each signal gives the difference of longitude between the two stations except for the error due to the time of transmission between the stations, which is eliminated by averaging the recorded times of the signals sent both ways. The personal equation (Art. 52) may be eliminated

by exchanging observers, although the impersonal transit micrometer now used practically eliminates the necessity of interchange. The work of longitude determination in the United

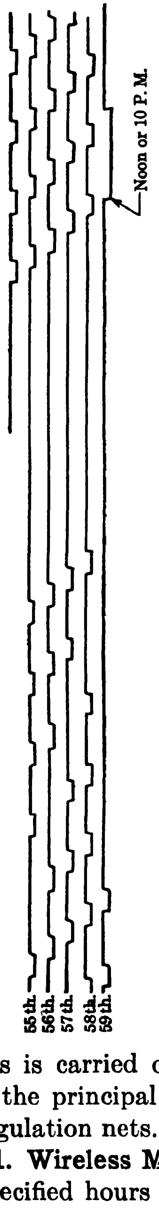


FIG. 74.—The chronograph record of the radio time signals of the U. S. Naval Radio Station showing 10 seconds of each of the last five minutes of the hour during which signals are sent. The last four second-beats of each minute are omitted. For the 55th minute the 51st second-beat is also omitted, for the 56th the 52nd, etc. For the 59th minute all the nine second-beats are omitted, the beginning of the next dash ndicating noon or 10 P.M., Eastern Standard Time.

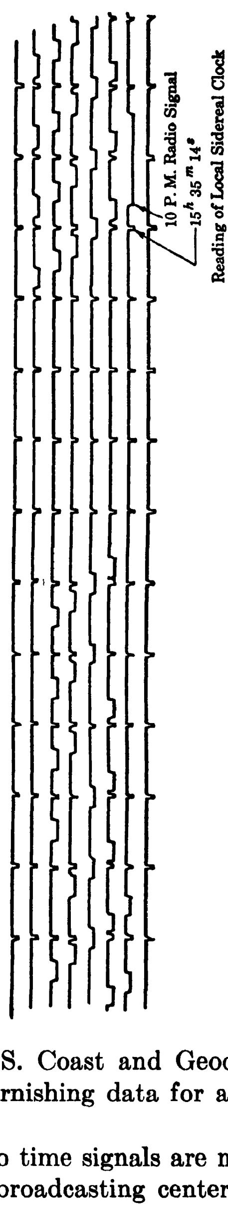


FIG. 75.—A portion of the chronographic record during a longitude determination. The local-clock beats are the short dashes while the longer dashes are the time signals from the U. S. Naval Station. The interval between seconds for the local clock is shorter than that for the time signals since the local clock keeps sidereal time and the clock sending out the time signals keeps mean.

States is carried on by the U. S. Coast and Geodetic Survey with the principal purpose of furnishing data for adjusting the triangulation nets.

121. Wireless Method.—Radio time signals are now sent out at specified hours from various broadcasting centers in Europe

and America. These signals, received at the station the longitude of which is required, are recorded on the chronograph along with the second-beats of the local clock or chronometer, either by ear and key or automatically by use of a suitable amplifier.[1] The difference between the time indicated by these signals and the local time determined as in the telegraphic method gives the required difference in longitude. There is only one field party, since there is no exchange of signals.

The program of star observations is the same as before, that is, one set of transits is observed before, and another after reception of the time signals. The approximate longitude of the station may be obtained from a map; this is necessary for computing the approximate local sidereal time at the station.

TIME SIGNALS
Domestic

Station	Location	Frequency, K.C.	Greenwich Civil Time of transmission
			Hours
NAA	Arlington, Va. Naval Radio Station	113 0 4,205.0	3, 8, 17 3, 17
NSS	Annapolis, Md. Naval Radio Station	17.8	3, 8, 17
NPG	Mare Island, Calif.	42.8 66 0 108 0	3, 8, 17 3, 8, 17 3, 8, 17

Foreign

FYL	Bordeaux, France Government Radio Station	15.7	8^h 6^m
GBR	Rugby, England Government Radio Station	16.0	18^h

The time signals, as usually sent by American stations, consist of short dashes every second beginning with the last five minutes

[1] For special apparatus see "Wireless Longitude," *Special Publication* 109, U. S. Coast and Geodetic Survey.

of the hour. In each minute the 29th second-beat and the 56th to 59th inclusive, are omitted. To indicate the minute, some other second-beats are also omitted between the 51st and 55th second. Figure 74 shows the signals for the last ten seconds of each minute. The last dash for the 60th second of the last minute is longer. Some European stations send out vernier signals. A list of stations with their broadcasting frequency and the Greenwich Civil Time of broadcast is given on p. 178. Signals sent out at 10:00 P.M. E.S.T. are listed 3^h G.C.T. of the following day. The time signals for all American stations listed above are based on time observations at the Naval Observatory. It is, of course, impossible to send these signals at exactly the time specified and a small correction, usually less than $0^s.1$, must be applied. The Naval Observatory distributes, at request, a list of corrections weekly.

Example: Compute the longitude of the Warner and Swasey Observatory from observations made on June 23, 1930. The Arlington 10 P.M. (75th meridian) time signals were recorded automatically on the chronograph, together with the beats of the local sidereal clock (Fig. 75).

	h	m	s
Clock reading of 10 P.M., radio time signal (average of 10 breaks used)	15	35	14.31
Clock correction at 10 P.M. (determined by star observations)			+43.60
Receiving apparatus lag (approximate)			−0 02
Time of transmission of signals			−0.00
Local time of 10 P.M. (75th meridian)	15	35	57.89
Washington sidereal time of 10 P.M., 75th meridian (communicated from the Naval Observatory)	15	53	58.55
Longitude of the Warner and Swasey Observatory, west of Washington		18	0 66

CHAPTER XIV

THE ZENITH TELESCOPE

122. The Principle of the Zenith Telescope.—The meridian zenith distance of a star south of the zenith (Fig. 76) is

$$z' = \phi - \delta' \tag{99}$$

and that of a star north of the zenith and above the pole is

$$z'' = \delta'' - \phi. \tag{100}$$

From which we obtain

$$\phi = \tfrac{1}{2}(\delta' + \delta'') + \tfrac{1}{2}(z' - z''). \tag{101}$$

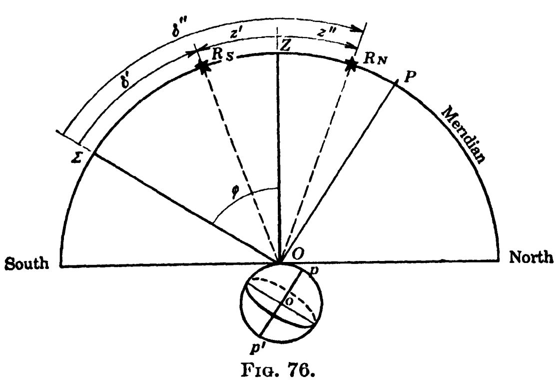


FIG. 76.

This equation suggests a method of determining the latitude by observing the small difference in the meridian zenith distances of two stars of known declination culminating at about the same time and on opposite sides of the zenith. It is usually known as the Horrebow-Talcott method.

123. The zenith telescope is an instrument designed to measure differences in zenith distance. The type of instrument shown in Fig. 77 consists of a telescope T, attached, at right angles, to one end of a horizontal axis which rests on a vertical axis V. Two sensitive levels, L and L', known as the *latitude levels* are attached to a graduated circle C which is fixed to the telescope. The levels are used to *measure accurately* small deviations in

zenith distance and the circle reads zenith distance. The horizontal axis coincides with the east and west line so when the telescope is rotated about it the line of sight describes the meridian. The meridian may also be described on the other side of the vertical axis by simply reversing the entire instrument about the vertical axis. Therefore, when a certain star is observed south of the zenith, by reversing the instrument about the vertical axis and *without resetting the telescope*, another star of approximately the *same zenith distance* and *north* of the zenith, may be observed (Fig. 79). The instrument is provided with a horizontal circle at the foot of the vertical axis and a striding level to rest on the horizontal axis, both used when the adjustments are made.

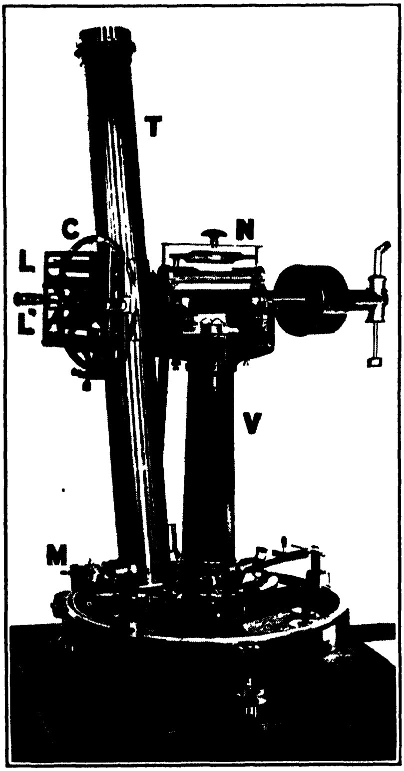


FIG. 77.—A zenith telescope. At the end of the telescope tube there is placed a right-angle prism to direct the rays of light horizontally through a side opening in the tube to the eyepiece at M. (*The Warner and Swasey Co.*)

124. Micrometer.—The eye end of the zenith telescope is fitted with a micrometer which is designed to measure difference in zenith distance of stars. The micrometer M shown in Fig. 77 has two perpendicular fixed wires AA' and EE' (Fig. 78) placed in the focal plane of the objective. If the instrument is properly adjusted, AA' remains in the meridian as the telescope is rotated about the horizontal axis. The center of the field is marked by the intersection of AA' and EE'. In the same plane and perpendicular to AA' is a wire BB', moved by means of a graduated micrometer head H. The angular value of one turn of the screw is denoted by R and in this case it is equal to 42.879. The head is

graduated into 100 divisions to read the fractions of a turn; the whole turns are given by a graduated disk G connected with the head. Fifty turns will carry the wire across the field; therefore 50 times 42″.879 gives the extent of the field, *i.e.*, approximately 36 minutes of arc. When the moving wire is in the center of the field the micrometer reading is about 25. Usually other fixed wires are placed parallel to the meridian wire so that the instrument may be used for time observations (Chap. XII). Again, many transit instruments are fitted with a

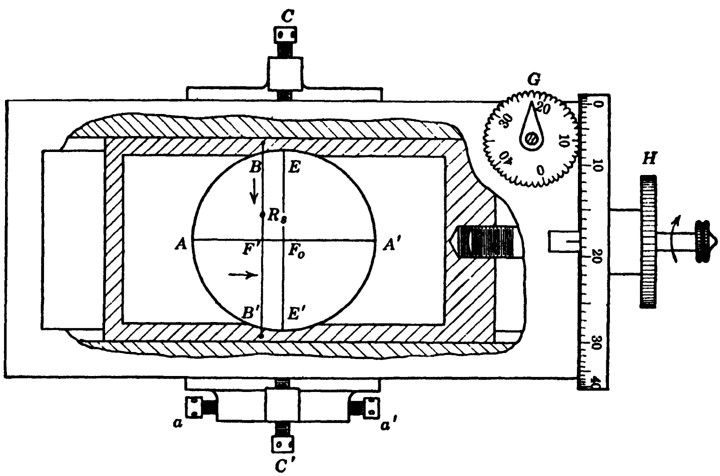


FIG. 78.—The star R_8 is bisected with the wire BB' by turning the micrometer head H. When the star comes to the meridian wire AA', the micrometer is read. In this case the reading is 22.184.

micrometer and with latitude levels so that they may be used for latitude determinations. The instruments shown in Figs. 67 and 68 are adapted for latitude determinations.

The method of determining latitude by the zenith telescope yields the best results for the following reasons:

1. It measures differences of zenith distance instead of absolute zenith distances.

2. All errors of graduation of circles are eliminated. The circle C simply furnishes the means for setting on a star.

3. Since the pair of stars used in the determination have very nearly the same meridian zenith distances and only the difference of these zenith distances enters the computation, the refraction effect is very small.

125. Adjustments of the Zenith Telescope.—The instrument rests on three foot-screws, two of which are set parallel to the east and west line.

a. To Make the Vertical Axis Truly Vertical.—Place the striding level N (Fig. 77) on the horizontal axis and rotate the instrument about the vertical. If the level bubble remains at the *same* position throughout, the axis is truly vertical; if not, adjust by means of the foot-screws. For the final adjustment the latitude levels may be used.

b. To Make the Horizontal Axis Perpendicular to the Vertical.—The position of this axis may be tested by reading the striding level both direct and reversed. The process of adjustment is the same as in the case of the horizontal axis of the transit instrument (Art. 112). Adjust by means of the screw just below and at one end of the horizontal axis.

c. To Make the Movable Micrometer Wire Horizontal.—With this wire bisect a distant well-defined object and place its image at one end of the field. Clamp the telescope in zenith distance in that position; slightly rotate it about the vertical axis and see if the object remains bisected; if not, adjust by whatever means are provided (Fig. 78, adjusting screws a and a'). It is well to check this adjustment by slow-moving stars after the other adjustments have been made. When the telescope is on the meridian, point to a circumpolar star (*i.e.*, a slow-moving star) and read the micrometer a number of times as the star moves over the field. The average of the readings on one side of the meridian should be the same as the average on the other side.

d. Collimation Adjustment: to Make the Line of Sight Perpendicular to the Horizontal Axis.—The line joining the intersection F_0 of the two fixed cross-wires with the center of the objective, is the *line of sight*. If the instrument is made so that the horizontal axis is reversible on its supports the adjustment is the same as in the transit instrument (Art. 112). If it is of the type shown in Fig. 77 two distant marks are placed side by side, with the distance between them equal to twice the distance between the vertical axis and the center of the telescope tube. Sight to the one mark, rotate the telescope 180° by means of the horizontal circle and sight on the second mark. If the intersection of the cross-wires does not bisect this mark, move the reticle toward the mark one quarter of the way with the adjusting screws provided for this purpose (Fig. 78, C and C'). Reverse

instrument and repeat process. The vertical wire EE' may easily be made truly vertical by pointing to a distant mark; bisect it at the upper edge of the field, move the telescope in zenith distance, and see if the bisection remains. This necessitates rechecking the collimation adjustment.

e. Azimuth Adjustment: to Make the Line of Sight Lie Always on the Meridian.—This is done by adjusting the stops of the horizontal circle and for this purpose the time must be known within a second. Follow a slow-moving star with the vertical wire by means of the tangent screw of the horizontal circle or the screw of the stop until the clock indicates that the star is on the meridian, and adjust the stop to this position. Repeat for the other stop with another star.

When the line of sight is vertical the graduated circle C must read zero. To do this bring the bubbles of the latitude levels approximately to the centers of their tubes and adjust the vernier of the circle to read zero.

It is very important that the eyepiece be carefully focused as the value of one turn of the micrometer changes with the focus.

126. The Observing List.—Since there are a number of requirements which a pair of stars to be observed must fulfill, the stars given in the American Ephemeris are not sufficient. Others may be obtained from more extensive catalogues. Boss' "Preliminary General Catalogue" lists 6,188 stars and is at present the best.

The sidereal time of observation being known, we choose a star having a R.A. corresponding to this time and a declination such that its zenith distance is less than 30°. From Eq. (101), we see that

$$(\delta' + \delta'') - 2\phi = (z'' - z'), \tag{102}$$

i.e., the sum of the declinations of the two stars minus twice the latitude is equal to the difference in the zenith distances. The two stars that form the pair must be chosen so that the difference in their zenith distances is less than the extent of the field of the telescope.

The difference in R.A. of the two stars must be at least 4 minutes in order to give sufficient time to complete the readings for the first star, and not more than 20 minutes, as the constants of the instrument might change during that interval. Stars fainter than eighth magnitude will be found difficult to observe.

To illustrate the above, consider the following:

Observing List

No.	Star	Magn.	Mean α, 1930	Mean δ, 1930	N or S	z	Setting	Micrometer setting
			h m s	° ′		° ′	° ′	
1	B.5153	4 7	20 02 31	67 40	N	26 08	26 21	18 down
	B.5182	5 0	20 11 02	14 59	S	26 33		
2	B.5208	5.1	20 15 12	37 49	S	3 43	3 53	14 down
	B 5230	5.9	20 19 48	45 34	N	4 02		
3	B.5267	6.5	20 28 20	36 42	S	4 50	4 53	4 down
	B.5283	5 8	20 31 36	46 27	N	4 55		

Column 2 gives the name or number from catalogue; in this case all the stars were taken from Boss' "Preliminary General Catalogue." Column 3 gives the magnitude, which serves to identify the stars. Columns 4 and 5 give the approximate R.A. and declination; these are the mean values (Art. 39) for the year of observation. The R.A. is given to the nearest second and indicates when the star is on the meridian; the declination is given to the nearest minute and serves to compute the zenith distance. Column 6 indicates whether the star will culminate north or south of the zenith. Column 7 is computed from Eqs. (99) and (100) using the best known value of the latitude of the place, which may be obtained from a map or determined by one of the methods given in Chap. IX. In this illustration it is taken as 41° 32′ N. Column 8 shows the average of the zenith distance of the stars in the pair.

To obtain the last column proceed as follows: (*a*) get the difference between the zenith distance of the first star of the pair and the "setting"; (*b*) reduce this difference to the corresponding number of turns of the micrometer. For example, in the case of the first pair, this difference is 13′ or 780″; the value of one turn of the micrometer is approximately 43″. Hence, the number of turns of the micrometer for this difference is 18. This column indicates the approximate position of the star in the field and helps to identify it. That is, if the movable micrometer wire is set 18 revolutions below the center of the field, the star will appear near this wire. When the setting is *less* than the zenith distance of the first star, the movable wire should be set

the indicated number of turns *above* the center of the field and when *greater*, *below*.

From the observing list we see:

a. All the stars used are brighter than the eighth magnitude.

b. The right ascensions in each pair do not differ by much less than 4 minutes nor more than 20.

c. The zenith distance of each star is less than 30°.

d. The difference of zenith distance in each pair is less than the extent of the field, which is 36′ or 50 revolutions of the micrometer head in the case under consideration.

127. Directions for Observing.—The vernier attached to the latitude levels is set to read the average zenith distance of the two stars (columns 6 and 8 of the observing list). It is evident that the first star will cross the field above the center and the second below or *vice versa; e.g.*, in the case of the first pair the first star will be observed about 18 revolutions below the center and the second 18 above. Set the movable wire so many revolutions above or below the center according to column 9. In our case when the micrometer reads 25, the movable wire is at the center of the field, so the micrometer should be set to read 7 for the first star and 43 for the second star of the pair. Bring the level bubbles approximately to the center and clamp the motion of the telescope in the vertical plane with the lower clamp. Two or three minutes before the time of transit, the first star will appear to move near and parallel to the movable wire. *Turn the micrometer head so that this wire bisects the star and see that it remains bisected when the star crosses the meridian.* The time of transit may be recorded and should correspond to the R.A. of the star; this gives assurance that the proper star was observed. On account of slight imperfections in adjustment of the instrument, this time might differ by a few seconds from the R.A. of the star; in such a case a correction may be applied to reduce the readings to the meridian, although it is very small and is usually omitted. This observed time may also differ by three or four seconds on account of the fact that the R.A. given in the observing list is the mean value. To complete the work on the first star *record the readings for the north and south ends of the latitude levels and for the micrometer.*

To observe the second star, reverse the telescope by rotating it about the vertical axis. If the latitude level bubbles are not in the center of their tubes, center them with the lower tangent

screw which moves the telescope. *Do not change the setting for the second star.* Place movable wire in a symmetrical position on the other side of the center of the field, *i.e.*, in the case under discussion 18 revolutions above the center; then proceed as with the first star.

The following should be kept in mind:

a. Do not change focus of the instrument.

b. Point telescope toward the first star with the head of the micrometer up (the reason for this will be made clear in the next article). Figure 77 shows the head upward.

c. Do not change "setting" until both stars of the set are observed.

d. Do not move micrometer wire back and forth in bisecting the star image as there is a certain amount of lost motion in the thread of the screw. Move it in the same direction for both stars.

128. Latitude Equation.—When the micrometer is inclined with the micrometer head upward, the greater the micrometer reading, the greater the zenith distance. It is therefore advisable *always to set the telescope so that the micrometer head is inclined upward,* as the formulas for reduction usually are based on that assumption. Figure 79 shows the head inclined upward.

Let

m_0 = the micrometer reading when the movable wire is in the middle of the field or, as a matter of fact, at any arbitrary position. When the movable wire is at F_0 (Fig. 79), the micrometer reads m_0.

z_0 = the zenith distance for the line of sight through F_0, as set at the circle C.

b = correction for level, positive when north reading is high, which indicates that the vertical axis is pointing south of the zenith. Thus, for stars south of the zenith, b is to be added to, and for stars north subtracted from the zenith distance z_0 [see Eqs. (103) and (104)].

m' = micrometer reading for south-of-zenith star, *i.e.*, when movable wire is at F'.

z' = the true zenith distance of the star south of zenith.

m'' = micrometer reading for north-of-zenith star, *i.e.*, when movable wire is at F''.

z'' = the true zenith distance of star north of zenith.

r = refraction correction.

R = angular value of one turn of micrometer head.

The angle $f' = (m' - m_0)R$ and $f'' = (m'' - m_0)R$.

Hence we may write for the star south of the zenith

$$z' = z_0 + (m' - m_0) \cdot R + b' + r' \tag{103}$$

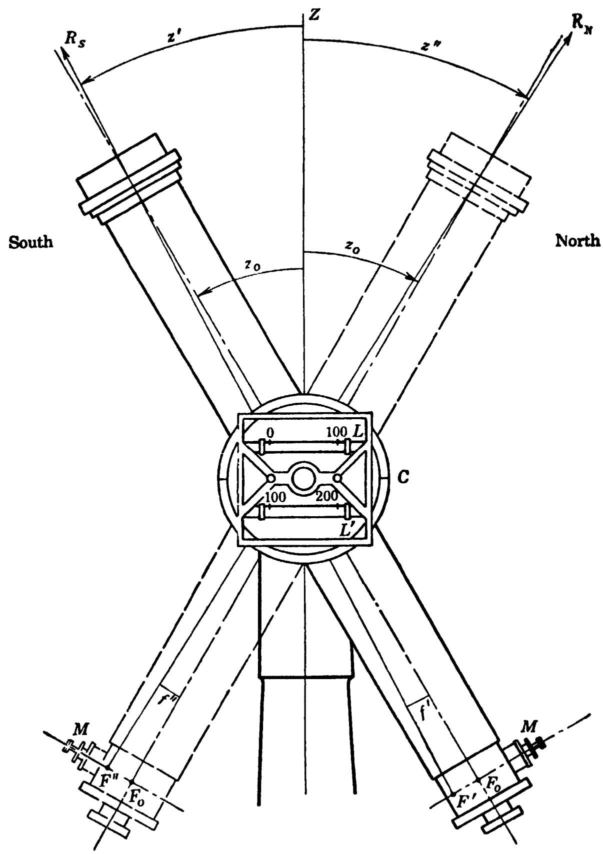


FIG. 79.—The average zenith distance, z_0, of the stars R_s and R_n is set on the circle C and the latitude levels centered. The star R_s having the smaller zenith distance (z') is observed below the center of the field at F'. The horizontal axis is then reversed so that the telescope will be pointing to the star R_n when it comes to the meridian; having the greater zenith distance (z'') it will appear above the center of the field at F''. The micrometer M measures the sum of the angles f' and f'' which sum is equivalent to the difference of the zenith distances of the two stars ($z'' - z'$).

and for a star north of the zenith

$$z'' = z_0 + (m'' - m_0) \cdot R - b'' + r''. \tag{104}$$

Therefore

$$z' - z'' = (m' - m'') \cdot R + b' + b'' + r' - r''.$$

Substituting in Eq. (101), we have:

$$\phi = \overbrace{\tfrac{1}{2}(\delta' + \delta'')}^{1} + \overbrace{\tfrac{1}{2}(m' - m'') \cdot R}^{2} + \overbrace{\tfrac{1}{2}(b' + b'')}^{3} + \overbrace{\tfrac{1}{2}(r' - r'')}^{4} \quad (105)$$

where the "primes" refer to stars south of the zenith and "seconds" to stars north.

1. *Declinations.*—If the stars are given in the American Ephemeris, obtain their apparent declinations from it (Art. 36). If they are given in other catalogues, it will be necessary first to reduce their coordinates to the mean place for the year of observation (Art. 39) and from this to the apparent place at the time of observation (Art. 40). The mean value of the right ascension for the year is sufficiently accurate. It is to be used as the sidereal time of the observation for the observing list and for the reduction of the apparent declination.

2. *The micrometer term* is equal to one-half of the difference in zenith distances of the two stars. If the readings were made with the micrometer head down the sign of the term must be changed.

3. *Level Correction.*—Let n' and s' be the north and south readings of the level for the south star and n'' and s'' for the north star, d the value of one division of the level in seconds of arc, and x the error of the level. Then,

a. If the graduations of the level are numbered in both directions from the middle,

$$b' = \tfrac{1}{2}(n' - s') \cdot d + x$$
$$b'' = \tfrac{1}{2}(n'' - s'') \cdot d - x$$

and

$$\tfrac{1}{2}(b' + b'') = \tfrac{1}{4}[(n' - s') + (n'' - s'')] \cdot d. \quad (106)$$

b. If the graduations of the level are numbered continuously from one end to the other with the numbers increasing toward the eyepiece (when the telescope is inclined)

$$\tfrac{1}{2}(b' + b'') = \tfrac{1}{4}\,[(n' + s') - (n'' + s'')] \cdot d. \quad (107)$$

c. If the graduations are numbered from one end but with the numbers increasing toward the objective (when the telescope is inclined)

$$\tfrac{1}{2}(b' + b'') = \tfrac{1}{4}\,[(n'' + s'') - (n' + s')] \cdot d. \quad (108)$$

Figure 79 shows the graduations of the level increasing toward the eyepiece so that if all the observations are made with the

head of the micrometer up, Eq. (107) is to be used for the level correction. To increase the accuracy of the level correction many instruments are provided with two levels. The value of the correction is obtained either directly, that is, by averaging the correction of the two levels, or by averaging the corresponding level readings and substituting these in the suitable formula.

4. *Refraction correction* being a function of the difference in the zenith distances is always very small.

From Eq. (32) we have

$$r' - r'' = 60''.6 \ (\tan z' - \tan z'') = 60''.6 \frac{\sin (z' - z'')}{\cos z' \cos z''}.$$

Or, since z' and z'' are nearly equal to z_0

$$\tfrac{1}{2}(r' - r'') = 30''.3 \frac{\sin (z' - z'')}{\cos^2 z_0}. \tag{109}$$

Table XII gives the value of this correction; since half the difference in zenith distance is equivalent to the second term of Eq. (105) and is already computed, the table is prepared for $\frac{1}{2}(z' - z'')$. The average of the zenith distances (z_0) may be obtained from the observing list. The sign of the correction is the same as the sign of $\frac{1}{2}(z' - z'')$.

Example: The following observations were made at the Warner and Swasey Observatory (lat. 41° 32′ N, long. 5^h 26^m W) on July 29, 1930:

$R = 42''.879$, $d = 0''.712$ for upper level.
$d = 0''.743$ for lower level.

OBSERVATIONS

Stars: B.5267, B.5283	Reference
Sidereal time of meridian transits: $20^h\ 28^m\ 17^s$; $20^h\ 31^m\ 39^s$	From Boss' "Preliminary General Catalogue"; see also Observing List, Art. 126.
Micrometer $m' = 22.184$. $m'' = 28.767$.	Observed

LEVELS

Star	Upper	Lower	Star	Upper	Lower
B.5267	$n' = 64.0$	159.0	B.5283	$n'' = 42.0$	129.0
	$s' = 40.5$	129.0		$s'' = 65.5$	159 5

REDUCTION

		°	′	″	Reference
	$\delta' =$	36	41	58 13	From mean place of Boss' "Preliminary General Catalogue," 1900 to epoch of 1930 (Art. 39). From this to the apparent place at the time of observation (Art. 40).
	$\delta'' =$	46	27	11.36	
1.	$\frac{1}{2}(\delta' + \delta'') =$	41	34	34.75	
2.	$\frac{1}{2}(m' - m'') \cdot R =$		−2	21.14	
3.	$\frac{1}{2}(b' + b'') =$		−	0.32	Eq. (107)
4.	$\frac{1}{2}(r' - r'') =$		−	0.04	Table XII, using $\frac{1}{2}(z' - z'') = 2'\ 21.14$ and $z_0 = 4°53'$ (observing list)
	$\phi =$	41	32	13 25	Eq. (105)

Exercise

Latitude Determination.—The following observations were made at the Warner and Swasey Observatory: latitude 41° 32′ N, longitude $5^h\ 26^m$ W, on July 29, 1930.

Value of $R = 42''879$, $d = 0''712$ for upper level.
$d = 0.743$ for lower level.

Star	Micrometer	Levels			
		Upper	Lower	Upper	Lower
B.5208	$m' = 11.500$	$n' = 49\ 5$,	146.0,	$s' = 26\ 0$	115.5
B.5230	$m'' = 37.312$	$n'' = 29.5$,	118.0,	$s'' = 53.0$,	148.5

These stars are given in the observing list of Art. 126. The apparent declinations of the stars at the time of observation were:

$$\delta' = 37°\ \ 48'\ \ 53''44.$$
$$\delta'' = 45\ \ \ 34\ \ \ 10.56.$$

CONSTANTS AND FORMULAS

Note 1.

	π	3.1415927
	$\frac{1}{\pi}$	0.3183099
One radian............	$\frac{360°}{2\pi}$	57.29578
One radian.......	$\frac{21600'}{2\pi}$	3437′747
One radian......... ...	$\frac{1296000''}{2\pi}$	206264″8
One degree of arc......	$\frac{\pi}{180}$	0 174533 radian
One minute of arc... .	$\frac{\pi}{10800}$	0 0002909 radian
One second of arc.....	$\frac{\pi}{648000}$	0 00000485 radian

Note 2.

If x is a small angle we may write

$$\sin x = x \text{ (in radians)}$$
$$\sin x = x' \sin 1'$$
$$\sin x = x'' \sin 1''$$

$$\sin 1' = 0\ 00029089$$
$$\sin 1'' = 0.00000485$$
$$\log \sin 1'' = 4.6855749 - 10$$

$$\tan x = x \text{ (in radians)}$$
$$\tan x = x' \tan 1'$$
$$\tan x = x'' \tan 1''$$

$$\tan 1' = 0\ 00029089$$
$$\tan 1'' = 0.00000485$$
$$\log \tan 1'' = 4.6855749 - 10$$

Note 3.

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \cdots$$

$$(1 + x)^{-1} = 1 - x + x^2 - x^3 + \cdots$$

$$(1 - x)^{-1} = 1 + x + x^2 + x^3 + \cdots$$

TABLES

Table I.—Degrees, Minutes, Seconds of Arc into Hours, Minutes, Seconds of Time

Degrees						Minutes						Seconds			
°	h	m	°	h	m	′	m	s	′	m	s	″	s	″	s
0	0	00	30	2	00	0	0	00	30	2	00	0	0 00	30	2.00
1	0	04	31	2	04	1	0	04	31	2	04	1	0 07	31	2 07
2	0	08	32	2	08	2	0	08	32	2	08	2	0 13	32	2 13
3	0	12	33	2	12	3	0	12	33	2	12	3	0 20	33	2.20
4	0	16	34	2	16	4	0	16	34	2	16	4	0.27	34	2.27
5	0	20	35	2	20	5	0	20	35	2	20	5	0 33	35	2.33
6	0	24	36	2	24	6	0	24	36	2	24	6	0 40	36	2 40
7	0	28	37	2	28	7	0	28	37	2	28	7	0 47	37	2.47
8	0	32	38	2	32	8	0	32	38	2	32	8	0 53	38	2.53
9	0	36	39	2	36	9	0	36	39	2	36	9	0.60	39	2 60
10	0	40	40	2	40	10	0	40	40	2	40	10	0 67	40	2.67
11	0	44	41	2	44	11	0	44	41	2	44	11	0 73	41	2 73
12	0	48	42	2	48	12	0	48	42	2	48	12	0 80	42	2 80
13	0	52	43	2	52	13	0	52	43	2	52	13	0 87	43	2.87
14	0	56	44	2	56	14	0	56	44	2	56	14	0 93	44	2 93
15	1	00	45	3	00	15	1	00	45	3	00	15	1 00	45	3 00
16	1	04	46	3	04	16	1	04	46	3	04	16	1 07	46	3 07
17	1	08	47	3	08	17	1	08	47	3	08	17	1.13	47	3 13
18	1	12	48	3	12	18	1	12	48	3	12	18	1 20	48	3 20
19	1	16	49	3	16	19	1	16	49	3	16	19	1 27	49	3.27
20	1	20	50	3	20	20	1	20	50	3	20	20	1 33	50	3.33
21	1	24	51	3	24	21	1	24	51	3	24	21	1 40	51	3.40
22	1	28	52	3	28	22	1	28	52	3	28	22	1 47	52	3.47
23	1	32	53	3	32	23	1	32	53	3	32	23	1.53	53	3 53
24	1	36	54	3	36	24	1	36	54	3	36	24	1.60	54	3 60
25	1	40	55	3	40	25	1	40	55	3	40	25	1 67	55	3.67
26	1	44	56	3	44	26	1	44	56	3	44	26	1 73	56	3.73
27	1	48	57	3	48	27	1	48	57	3	48	27	1 80	57	3 80
28	1	52	58	3	52	28	1	52	58	3	52	28	1 87	58	3 87
29	1	56	59	3	56	29	1	56	59	3	56	29	1.93	59	3.93
30	2	00	60	4	00	30	2	00	60	4	00	30	2.00	60	4.00

TABLE II.—SIDEREAL INTO MEAN SOLAR TIME

Mean time interval (I) = sidereal time interval (I') − C

Sidereal hours	C		Sidereal minutes	C		Sidereal minutes	C		Sidereal seconds	C	Sidereal seconds	C
	m	s		m	s		m	s		s		s
0	0	000	0	0	000	31	0	5 079	0	0 000	31	0 085
1	0	9 830	1	0	0 164	32	0	5 242	1	0 003	32	0 087
2	0	19 659	2	0	0 328	33	0	5 406	2	0 005	33	0 090
3	0	29 489	3	0	0 491	34	0	5 570	3	0 008	34	0 093
4	0	39 318	4	0	0 655	35	0	5 734	4	0 011	35	0 096
5	0	49 148	5	0	0 819	36	0	5 898	5	0 014	36	0 098
6	0	58 977	6	0	0 983	37	0	6 062	6	0 016	37	0 101
7	1	8 807	7	0	1 147	38	0	6 225	7	0 019	38	0 104
8	1	18 636	8	0	1 311	39	0	6 389	8	0 022	39	0 106
9	1	28 466	9	0	1.474	40	0	6 553	9	0 025	40	0 109
10	1	38 296	10	0	1 638	41	0	6 717	10	0 027	41	0 112
11	1	48 125	11	0	1 802	42	0	6 881	11	0 030	42	0.115
12	1	57.955	12	0	1 966	43	0	7 045	12	0.033	43	0 117
13	2	7.784	13	0	2 130	44	0	7 208	13	0 035	44	0 120
14	2	17 614	14	0	2 294	45	0	7 372	14	0.038	45	0.123
15	2	27 443	15	0	2 457	46	0	7 536	15	0.041	46	0 126
16	2	37 273	16	0	2 621	47	0	7 700	16	0.044	47	0.128
17	2	47 102	17	0	2 785	48	0	7 864	17	0.046	48	0.131
18	2	56 932	18	0	2 949	49	0	8 027	18	0 049	49	0 134
19	3	6 762	19	0	3 113	50	0	8.191	19	0 052	50	0.137
20	3	16 591	20	0	3 277	51	0	8.355	20	0 055	51	0.139
21	3	26 421	21	0	3.440	52	0	8 519	21	0 057	52	0.142
22	3	36 250	22	0	3 604	53	0	8 683	22	0 060	53	0 145
23	3	46 080	23	0	3 768	54	0	8 847	23	0.063	54	0 147
24	3	55 909	24	0	3 932	55	0	9.010	24	0 066	55	0.150
			25	0	4.096	56	0	9.174	25	0 068	56	0 153
			26	0	4.259	57	0	9.338	26	0.071	57	0 156
			27	0	4.423	58	0	9 502	27	0.074	58	0 158
			28	0	4 587	59	0	9 666	28	0.076	59	0 161
			29	0	4 751	60	0	9 830	29	0 079	60	0.164
			30	0	4 915				30	0 082		

Table III.—Mean Solar into Sidereal Time

Sidereal time interval (I') = mean time interval (I) + C

Mean hours	C	Mean minutes	C	Mean minutes	C	Mean seconds	C	Mean seconds	C
	m s		m s		m s		s		s
0	0 000	0	0 000	31	0 5 093	0	0 000	31	0 085
1	0 9 856	1	0 0 164	32	0 5 257	1	0 003	32	0 088
2	0 19 713	2	0 0 329	33	0 5 421	2	0 005	33	0 090
3	0 29 569	3	0 0 493	34	0 5 585	3	0 008	34	0 093
4	0 39 426	4	0 0 657	35	0 5 750	4	0 011	35	0 096
5	0 49 282	5	0 0 821	36	0 5 914	5	0 014	36	0.099
6	0 59.139	6	0 0 986	37	0 6 078	6	0 016	37	0 101
7	1 8 995	7	0 1 150	38	0 6 242	7	0 019	38	0 104
8	1 18.852	8	0 1 314	39	0 6 407	8	0 022	39	0 107
9	1 28 708	9	0 1 478	40	0 6 571	9	0.025	40	0 110
10	1 38 565	10	0 1 643	41	0 6 735	10	0 027	41	0 112
11	1 48 421	11	0 1 807	42	0 6 900	11	0 030	42	0.115
12	1 58 278	12	0 1 971	43	0 7 064	12	0 033	43	0 118
13	2 8 134	13	0 2 136	44	0 7 228	13	0 036	44	0 120
14	2 17 991	14	0 2 300	45	0 7 392	14	0 038	45	0 123
15	2 27 847	15	0 2 464	46	0 7 557	15	0 041	46	0 126
16	2 37 704	16	0 2 628	47	0 7 721	16	0 044	47	0 129
17	2 47 560	17	0 2 793	48	0 7 885	17	0 047	48	0 131
18	2 57 417	18	0 2 957	49	0 8 049	18	0 049	49	0 134
19	3 7 273	19	0 3 121	50	0 8 214	19	0 052	50	0.137
20	3 17.129	20	0 3 285	51	0 8 378	20	0 055	51	0 140
21	3 26 986	21	0 3 450	52	0 8 542	21	0 057	52	0.142
22	3 36 842	22	0 3 614	53	0 8 707	22	0 060	53	0 145
23	3 46 699	23	0 3 778	54	0 8.871	23	0 063	54	0 148
24	3 56 555	24	0 3 943	55	0 9 035	24	0 066	55	0.151
		25	0 4 107	56	0 9 199	25	0 068	56	0.153
		26	0 4 271	57	0 9 364	26	0 071	57	0 156
		27	0 4 435	58	0 9.528	27	0 074	58	0 159
		28	0 4 600	59	0 9.692	28	0 077	59	0 162
		29	0 4.764	60	0 9 856	29	0 079	60	0 164
		30	0 4 928			30	0 082		

TABLE IV.—MEAN REFRACTION
(For a temperature of 50° F. and barometric pressure of 29.6 in.)

Altitude	Zenith distance	Refraction		Altitude	Zenith distance	Refraction	
°	°	′	″			′	″
0	90	34	50	18	72	2	56
1	89	24	22	20	70	2	37
2	88	18	06	22	68	2	21
3	87	14	13	24	66	2	08
4	86	11	37	26	64	1	57
5	85	9	45	28	62	1	47
6	84	8	23	30	60	1	39
7	83	7	19	35	55	1	22
8	82	6	29	40	50	1	08
9	81	5	49	45	45	0	57
10	80	5	16	50	40	0	48
11	79	4	48	55	35	0	40
12	78	4	25	60	30	0	33
13	77	4	04	65	25	0	27
14	76	3	47	70	20	0	21
15	75	3	33	75	15	0	15
16	74	3	18	80	10	0	10
17	73	3	07	85	5	0	05
18	72	2	56	90	0	0	00

TABLE V.—PARALLAX AND SEMIDIAMETER OF THE SUN

A		*B*		
Zenith distance	Parallax	Date	Semidiameter	
°	″		′	″
0	0	Jan. 1	16	18
10	2	Feb. 1	16	16
20	3	March 1	16	10
30	4	April 1	16	02
40	6	May 1	15	54
50	7	June 1	15	48
60	8	July 1	15	46
70	8	Aug. 1	15	47
80	9	Sept. 1	15	53
90	9	Oct. 1	16	00
		Nov. 1	16	09
		Dec. 1	16	15

TABLE VI.—REDUCTION TO THE MERIDIAN*

$$m = \frac{2 \sin^2 \frac{1}{2}t}{\sin 1''}$$

t	0^m	1^m	2^m	3^m	4^m	5^m	6^m	7^m	8^m
s	″	″	″	″	″	″	″	″	″
0	0 00	1 96	7 85	17 67	31 42	49 09	70 68	96 20	125 65
1	0 00	2 03	7 98	17 87	31 68	49 41	71 07	96 66	126 17
2	0 00	2 10	8 12	18 07	31 94	49 74	71 47	97 12	126 70
3	0.00	2 16	8 25	18 27	32 20	50 07	71 86	97 58	127 22
4	0.01	2.23	8 39	18.47	32.47	50 40	72 26	98 04	127 75
5	0 01	2 31	8.52	18.67	32 74	50 73	72 66	98 50	128 28
6	0 02	2 38	8 66	18 87	33.01	51 07	73 06	98 97	128 81
7	0 02	2 45	8.80	19 07	33.27	51 40	73 46	99 43	129 34
8	0 03	2 52	8 94	19 28	33 54	51 74	73 86	99 90	129 87
9	0.04	2 60	9.08	19 48	33.81	52.07	74.26	100 37	130 40
10	0 05	2 67	9 22	19 69	34 09	52 41	74 66	100 84	130 94
11	0.06	2 75	9 36	19 90	34 36	52 75	75 06	101 31	131 47
12	0 08	2 83	9 50	20.11	34 64	53 09	75 47	101 78	132 01
13	0.09	2 91	9 64	20.32	34.91	53 43	75 88	102 25	132 55
14	0.11	2 99	9 79	20.53	35.19	53.77	76 29	102 72	133 09
15	0.12	3.07	9 94	20 74	35 46	54 11	76.69	103 20	133 63
16	0.14	3 15	10 09	20.95	35 74	54 46	77 10	103 67	134 17
17	0 16	3 23	10 24	21.16	36 02	54 80	77 51	104 15	134 71
18	0 18	3 32	10 39	21.38	36 30	55.15	77 93	104 63	135 25
19	0.20	3 40	10.54	21.60	36.58	55.50	78.34	105 10	135 80
20	0 22	3 49	10.69	21 82	36 87	55 84	78 75	105 58	136 34
21	0 24	3 58	10 84	22 03	37 15	56 19	79 16	106 06	136 88
22	0 26	3 67	11 00	22 25	37 44	56 55	79 58	106 55	137 43
23	0 28	3 76	11 15	22 47	37.72	56 90	80 00	107 03	137 98
24	0 31	3 85	11 31	22.70	38 01	57.25	80 42	107.51	138 53
25	0 34	3 94	11 47	22 92	38 30	57.60	80 84	107 99	139 08
26	0.37	4 03	11.63	23 14	38 59	57 96	81 26	108 48	139 63
27	0 40	4 12	11 79	23.37	38 88	58 32	81 68	108.97	140 18
28	0 43	4 22	11 95	23 60	39.17	58 68	82 10	109 46	140 74
29	0 46	4 32	12 11	23.82	39.46	59 03	82 52	109 95	141 29
30	0 49	4 42	12 27	24 05	39.76	59 40	82 95	110 44	141 85
31	0 52	4 52	12 43	24 28	40 05	59 75	83 38	110 93	142 40
32	0 56	4 62	12 60	24 51	40 35	60 11	83 81	111 43	142 96
33	0.59	4 72	12 76	24 74	40 65	60 47	84 23	111 92	143 52
34	0.63	4.82	12 93	24 98	40 95	60.84	84.66	112.41	144 08
35	0 67	4 92	13 10	25 21	41 25	61.20	85 09	112.90	144 64
36	0.71	5 03	13 27	25 45	41 55	61 57	85 52	113 40	145 20
37	0 75	5 13	13 44	25 68	41.85	61 94	85 95	113 90	145 76
38	0.79	5 24	13.62	25 92	42.15	62 31	86 39	114 40	146 33
39	0.83	5 34	13.79	26.16	42.45	62.68	86.82	114 90	146.89
40	0.87	5.45	13 96	26 40	42 76	63.05	87.26	115 40	147 46
41	0.91	5 56	14 13	26 64	43 06	63 42	87 70	115 90	148 03
42	0 96	5 67	14.31	26 88	43 37	63 79	88 14	116 40	148.60
43	1.01	5 78	14.49	27 12	43.68	64 16	88 57	116 90	149 17
44	1.06	5 90	14.67	27 37	43.99	64 54	89 01	117 41	149 74
45	1 10	6.01	14.85	27 61	44.30	64.91	89 45	117 92	150 31
46	1 15	6.13	15.03	27 86	44 61	65 29	89 89	118 43	150 88
47	1 20	6.24	15.21	28 10	44 92	65 67	90 33	118 94	151 45
48	1 26	6.36	15.39	28 35	45 24	66 05	90 78	119 45	152 03
49	1.31	6.48	15.57	28.60	45.55	66 43	91.23	119 96	152 61
50	1 36	6.60	15.76	28 85	45.87	66 81	91 68	120 47	153 19
51	1 42	6.72	15 95	29 10	46.18	67 19	92 12	120 98	153 77
52	1 48	6.84	16.14	29 36	46 50	67 58	92 57	121 49	154 35
53	1.53	6.96	16.32	29 61	46 82	67 96	93 02	122 01	154 93
54	1.59	7.09	16.51	29.86	47.14	68.35	93 47	122 53	155 51
55	1.65	7.21	16.70	30 12	47.46	68.73	93 92	123 05	156 09
56	1.71	7 34	16.89	30 38	47 79	69.12	94 38	123 57	156 67
57	1.77	7 46	17.08	30.64	48 11	69 51	94 83	124 09	157 25
58	1 83	7 60	17 28	30.90	48 43	69 90	95 29	124 61	157 84
59	1 89	7 72	17 47	31 16	48 76	70 29	95 74	125 13	158 43

* From Chauvenet's "Spherical and Practical Astronomy."

TABLE VI.—REDUCTION TO THE MERIDIAN.—(*Continued*)

$$m = \frac{2\sin^2 \frac{1}{2}t}{\sin 1''}$$

t	9^m	10^m	11^m	12^m	13^m	14^m	15^m	16^m
s	″	″	″	″	″	″	″	″
0	159.02	196.32	237.54	282.68	331.74	384.74	441.63	502.46
1	159.61	196.97	238.26	283.47	332.59	385.65	442.62	503.50
2	160.20	197.63	238.98	284.26	333.44	386.56	443.60	504.55
3	160.80	198.28	239.70	285.04	334.29	387.48	444.58	505.60
4	161.39	198.94	240.42	285.83	335.15	388.40	445.56	506.65
5	161.98	199.60	241.14	286.62	336.00	389.32	446.55	507.70
6	162.58	200.26	241.87	287.41	336.86	390.24	447.54	508.76
7	163.17	200.92	242.60	288.20	337.72	391.16	448.53	509.81
8	163.77	201.59	243.33	289.00	338.58	392.09	449.51	510.86
9	164.37	202.25	244.06	289.79	339.44	393.01	450.50	511.92
10	164.97	202.92	244.79	290.58	340.30	393.94	451.50	512.98
11	165.57	203.58	245.52	291.38	341.16	394.86	452.49	514.03
12	166.17	204.25	246.25	292.18	342.02	395.79	453.48	515.09
13	166.77	204.92	246.98	292.98	342.88	396.72	454.48	516.15
14	167.37	205.59	247.72	293.78	343.75	397.65	455.47	517.21
15	167.97	206.26	248.45	294.58	344.62	398.58	456.47	518.27
16	168.58	206.93	249.19	295.38	345.49	399.52	457.47	519.34
17	169.19	207.60	249.93	296.18	346.36	400.45	458.47	520.40
18	169.80	208.27	250.67	296.99	347.23	401.38	459.47	521.47
19	170.41	208.94	251.41	297.79	348.10	402.32	460.47	522.53
20	171.02	209.62	252.15	298.60	348.97	403.26	461.47	523.60
21	171.63	210.30	252.89	299.40	349.84	404.20	462.48	524.67
22	172.24	210.98	253.63	300.21	350.71	405.14	463.48	525.74
23	172.85	211.66	254.37	301.02	351.58	406.08	464.48	526.81
24	173.47	212.34	255.12	301.83	352.46	407.02	465.49	527.89
25	174.08	213.02	255.87	302.64	353.34	407.96	466.50	528.96
26	174.70	213.70	256.62	303.46	354.22	408.90	467.51	530.03
27	175.32	214.38	257.37	304.27	355.10	409.84	468.52	531.11
28	175.94	215.07	258.12	305.09	355.98	410.79	469.53	532.18
29	176.56	215.75	258.87	305.90	356.86	411.73	470.54	533.26
30	177.18	216.44	259.62	306.72	357.74	412.68	471.55	534.33
31	177.80	217.12	260.37	307.54	358.62	413.63	472.57	535.41
32	178.43	217.81	261.12	308.36	359.51	414.59	473.58	536.50
33	179.05	218.50	261.88	309.18	360.39	415.54	474.60	537.58
34	179.68	219.19	262.64	310.00	361.28	416.49	475.62	538.67
35	180.30	219.88	263.39	310.82	362.17	417.44	476.64	539.75
36	180.93	220.58	264.15	311.65	363.07	418.40	477.65	540.83
37	181.56	221.27	264.91	312.47	363.96	419.35	478.67	541.91
38	182.19	221.97	265.68	313.30	364.85	420.31	479.70	543.00
39	182.82	222.66	266.44	314.12	365.75	421.27	480.72	544.09
40	183.46	223.36	267.20	314.95	366.64	422.23	481.74	545.18
41	184.09	224.06	267.96	315.78	367.53	423.19	482.77	546.27
42	184.72	224.76	268.73	316.61	368.42	424.15	483.79	547.36
43	185.35	225.46	269.49	317.44	369.31	425.11	484.82	548.45
44	185.99	226.16	270.26	318.27	370.21	426.07	485.85	549.55
45	186.63	226.86	271.02	319.10	371.11	427.04	486.88	550.64
46	187.27	227.57	271.79	319.94	372.01	428.01	487.91	551.73
47	187.91	228.27	272.56	320.78	372.91	428.97	488.94	552.83
48	188.55	228.98	273.34	321.62	373.82	429.93	489.97	553.93
49	189.19	229.68	274.11	322.45	374.72	430.90	491.01	555.03
50	189.83	230.39	274.88	323.29	375.62	431.87	492.05	556.13
51	190.47	231.10	275.65	324.13	376.52	432.84	493.08	557.24
52	191.12	231.81	276.43	324.97	377.43	433.82	494.12	558.34
53	191.76	232.52	277.20	325.81	378.34	434.79	495.15	559.44
54	192.41	233.24	277.98	326.66	379.26	435.76	496.19	560.55
55	193.06	233.95	278.76	327.50	380.17	436.73	497.23	561.65
56	193.71	234.67	279.55	328.35	381.08	437.71	498.28	562.76
57	194.36	235.38	280.33	329.19	381.99	438.69	499.32	563.87
58	195.01	236.10	281.12	330.04	382.90	439.67	500.37	564.98
59	195.66	236.82	281.90	330.89	383.82	440.65	501.41	566.08

TABLE VII.—FOR THE YEAR 1930 AND FOR LATITUDE 40° N

t	a	Difference	b	Difference	t	a	Difference	b	Difference
h	′		′		h	′		′	
0	−0		+65		12	+0		−63	
		22		2			21		2
1	−22		+63		13	+21		−61	
		20		7			20		6
2	−42		+56		14	+41		−55	
		18		10			18		10
3	−60		+46		15	+59		−45	
		13		13			13		14
4	−73		+33		16	+72		−31	
		8		16			9		15
5	−81		+17		17	+81		−16	
		3		16			3		17
6	−84		+1		18	+84		+1	
		3		17			3		16
7	−81		−16		19	+81		+17	
		9		15			8		16
8	−72		−31		20	+73		+33	
		13		14			13		13
9	−59		−45		21	+60		+46	
		18		10			18		10
10	−41		−55		22	+42		+56	
		20		6			20		7
11	−21		−61		23	+22		+63	
		21		2			22		2
12	−0		−63		24	+0		+65	

The refraction is included in the value of b.

TABLE VIII.—F_1

ϕ	1930	1940	1950	1960
°				
20	0.82	0.77	0.74	0 70
30	0.89	0 84	0.81	0 76
40	1.00	0 95	0 91	0.86
50	1.19	1.13	1.08	1.02
60	1.53	1.46	1.39	1.32

TABLE IX.—F_2, α, δ FOR POLARIS

Year	F_2	α	δ
		h m	° ′
1930	1 00	1 37	+88 56
1940	0 95	1 42	88 59
1950	0.91	1 48	89 02
1960	0 86	1 53	+89 05

TABLE X.—AZIMUTH OF POLARIS AT ELONGATION, 1930 TO 1940

Lat.	1930		1931		1932		1933		1934		1935		1936		1937		1938		1939		1940	
°	°	′	°	′	°	′	°	′	°	′	°	′	°	′	°	′	°	′	°	′	°	′
10	1	05 3	1	05 0	1	04 7	1	04 4	1	04 1	1	03 8	1	3 4	1	3 1	1	2 8	1	2 5	1	2 2
11	1	05 5	1	05 2	1	04 9	1	04 6	1	04 3	1	04 0	1	3 6	1	3.3	1	3 0	1	2 7	1	2 4
12	1	05 7	1	05 4	1	05 1	1	04 8	1	04 5	1	04 2	1	3.9	1	3.6	1	3 2	1	2 9	1	2 6
13	1	06 0	1	05 7	1	05 3	1	05 0	1	04 7	1	04 4	1	4.1	1	3 8	1	3.5	1	3 2	1	2 9
14	1	06 3	1	05 9	1	05 6	1	05 3	1	05 0	1	04 7	1	4 4	1	4.1	1	3 8	1	3 4	1	3 1
15	1	06.6	1	06 2	1	05 9	1	05 6	1	05 3	1	05 0	1	4.7	1	4 4	1	4.0	1	3 7	1	3 4
16	1	06.9	1	06 6	1	06 2	1	05 9	1	05 6	1	05 3	1	5.0	1	4.7	1	4.4	1	4.0	1	3 7
17	1	07 2	1	06 9	1	06 6	1	06 3	1	06 0	1	05.7	1	5 3	1	5.0	1	4.7	1	4.4	1	4.1
18	1	07 6	1	07.3	1	07 0	1	06 6	1	06 3	1	06 0	1	5 7	1	5.4	1	5.0	1	4.7	1	4.4
19	1	08 0	1	07 7	1	07 3	1	07 0	1	06 7	1	06 4	1	6 1	1	5.8	1	5.4	1	5.1	1	4 8
20	1	08 4	1	08 1	1	07 8	1	07 4	1	07 1	1	06 8	1	6 5	1	6.2	1	5.8	1	5 5	1	5 2
21	1	08 9	1	08 5	1	08 2	1	07.9	1	07 6	1	07 2	1	6 9	1	6.6	1	6 3	1	5 9	1	5.6
22	1	09 3	1	09 0	1	08 7	1	08 4	1	08 0	1	07 7	1	7.4	1	7 0	1	6.7	1	6 4	1	6 1
23	1	09 8	1	09 5	1	09 2	1	08 9	1	08 5	1	08 2	1	7.9	1	7 5	1	7 2	1	6.9	1	6 6
24	1	10 4	1	10 0	1	09.7	1	09 4	1	09 0	1	08 7	1	8.4	1	8.0	1	7.7	1	7.4	1	7 1
25	1	10 9	1	10 6	1	10 3	1	09 9	1	09 6	1	09 3	1	8 9	1	8.6	1	8.2	1	7.9	1	7 6
26	1	11 5	1	11 2	1	10 9	1	10 5	1	10 2	1	09 9	1	9 5	1	9 2	1	8.8	1	8 5	1	8 2
27	1	12 2	1	11.8	1	11 5	1	11 1	1	10 8	1	10 5	1	10 1	1	9 8	1	9.4	1	9.1	1	8.8
28	1	12 8	1	12 5	1	12 1	1	11 8	1	11.4	1	11 1	1	10.8	1	10 4	1	10.1	1	9.7	1	9 4
29	1	13.5	1	13 2	1	12 8	1	12 5	1	12 1	1	11 8	1	11.4	1	11 1	1	10.7	1	10.4	1	10 0
30	1	14.2	1	13 9	1	13 5	1	13 2	1	12.8	1	12.5	1	12.1	1	11.8	1	11.4	1	11.1	1	10.7

TABLE X.—AZIMUTH OF POLARIS AT ELONGATION, 1930 TO 1940.—(*Continued*)

Lat.	1930	1931	1932	1933	1934	1935	1936	1937	1938	1939	1940
°	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
31	1 15.0	1 14.6	1 14.3	1 13.9	1 13.6	1 13.2	1 12.9	1 12.5	1 12.2	1 11.8	1 11.5
32	1 15.8	1 15.4	1 15.1	1 14.7	1 14.4	1 14.0	1 13.7	1 13.3	1 13.0	1 12.6	1 12.2
33	1 16.7	1 16.3	1 15.9	1 15.6	1 15.2	1 14.9	1 14.5	1 14.1	1 13.8	1 13.4	1 13.0
34	1 17.5	1 17.2	1 16.8	1 16.4	1 16.1	1 15.7	1 15.4	1 15.0	1 14.6	1 14.3	1 13.9
35	1 18.5	1 18.1	1 17.7	1 17.4	1 17.0	1 16.6	1 16.3	1 15.9	1 15.5	1 15.2	1 14.8
36	1 19.5	1 19.1	1 18.7	1 18.3	1 18.0	1 17.6	1 17.2	1 16.8	1 16.5	1 16.1	1 15.7
37	1 20.5	1 20.1	1 19.7	1 19.4	1 19.0	1 18.6	1 18.2	1 17.8	1 17.5	1 17.1	1 16.7
38	1 21.6	1 21.2	1 20.8	1 20.4	1 20.0	1 19.7	1 19.3	1 18.9	1 18.5	1 18.1	1 17.7
39	1 22.7	1 22.3	1 21.9	1 21.6	1 21.2	1 20.8	1 20.4	1 20.0	1 19.6	1 19.2	1 18.8
40	1 23.9	1 23.5	1 23.1	1 22.7	1 22.3	1 22.0	1 21.6	1 21.2	1 20.8	1 20.4	1 20.0
41	1 25.2	1 24.8	1 24.4	1 24.0	1 23.6	1 23.2	1 22.8	1 22.4	1 22.0	1 21.6	1 21.2
42	1 26.5	1 26.1	1 25.7	1 25.3	1 24.9	1 24.5	1 24.1	1 23.7	1 23.2	1 22.8	1 22.4
43	1 27.9	1 27.5	1 27.1	1 26.7	1 26.3	1 25.8	1 25.4	1 25.0	1 24.6	1 24.2	1 23.8
44	1 29.4	1 29.0	1 28.5	1 28.1	1 27.7	1 27.3	1 26.8	1 26.4	1 26.0	1 25.6	1 25.2
45	1 30.9	1 30.5	1 30.1	1 29.6	1 29.2	1 28.8	1 28.3	1 27.9	1 27.5	1 27.1	1 26.6
46	1 32.5	1 32.1	1 31.7	1 31.2	1 30.8	1 30.4	1 29.9	1 29.5	1 29.1	1 28.6	1 28.2
47	1 34.3	1 33.8	1 33.4	1 32.9	1 32.5	1 32.0	1 31.6	1 31.2	1 30.7	1 30.3	1 29.8
48	1 36.1	1 35.6	1 35.2	1 34.7	1 34.3	1 33.8	1 33.4	1 32.9	1 32.5	1 32.0	1 31.5
49	1 38.0	1 37.5	1 37.1	1 36.6	1 36.1	1 35.7	1 35.2	1 34.8	1 34.3	1 33.8	1 33.4
50	1 40.0	1 39.5	1 39.1	1 38.6	1 38.1	1 37.7	1 37.2	1 36.7	1 36.2	1 35.8	1 35.3

In computing the above azimuths the mean declination of Polaris for the beginning of each year was used.

(The above table has been kindly furnished by *The United States Naval Observatory*.)

TABLE XI.—FOR REDUCING TO ELONGATION OBSERVATIONS MADE NEAR ELONGATION

Azimuth at elong. / * Time	1° 0′	1° 10′	1° 20′	1° 30′	1° 40′	1° 50′	2° 0′	2° 10′	Azimuth at elong. / Time *
m	″	″	″	″	″	″	″	″	m
0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0
1	0 0	0 0	0 0	+ 0 1	+ 0 1	+ 0 1	+ 0 1	+ 0 1	1
2	+ 0 1	+ 0 2	+ 0 2	0 2	0 2	0 3	0 3	0 3	2
3	0 3	0 4	0 4	0 5	0 5	0 6	0 6	0 7	3
4	0 5	0 6	0.7	0.8	0.9	1 0	1.1	1 2	4
5	+ 0 9	+ 1 0	+ 1 1	+ 1 3	+ 1 4	+ 1 6	+ 1 7	+ 1 9	5
6	1 2	1 4	1 6	1 8	2 1	2 3	2 5	2 7	6
7	1.7	2 0	2 2	2 5	2 8	3 1	3 4	3 7	7
8	2 2	2 6	2 9	3 3	3 7	4 0	4 4	4 8	8
9	2 8	3.2	3 7	4 2	4 6	5 1	5 6	6 0	9
10	+ 3 4	+ 4 0	+ 4 6	+ 5 1	+ 5 7	+ 6 3	+ 6 9	+ 7 4	10
11	4 1	4 8	5.5	6 2	6 9	7 6	8 3	9 0	11
12	4 9	5 8	6.6	7 4	8 2	9 0	9 9	10 7	12
13	5 8	6 8	7.7	8 7	9 7	10 6	11 6	12 6	13
14	6 7	7 8	9 0	10 1	11.2	12 3	13 4	14.6	14
15	+ 7 7	+ 9 0	+10 3	+11 6	+12 8	+14 1	+15 4	+16 7	15
16	8 8	10 2	11 7	13 2	14 6	16 1	17 5	19 0	16
17	9 9	11 5	13 2	14 9	16 5	18 2	19 8	21 5	17
18	11 1	12 9	14 8	16 7	18 5	20 4	22 2	24 1	18
19	12 4	14.4	16 5	18.6	20.6	22 7	24 7	26 8	19
20	+13 7	+16 0	+18 3	+20 6	+22 8	+25 1	+27 4	+29 7	20
21	15 1	17 6	20 1	22 7	25 2	27 7	30 2	32 7	21
22	16 6	19 3	22 1	24 9	27 6	30 4	33 2	35 9	22
23	18 1	21 1	24 2	27 2	30 2	33 2	36 2	39 3	23
24	19 7	23 0	26 3	29 6	32 9	36 2	39 5	42 8	24
25	+21 4	+25 0	+28 5	+32 1	+35 7	+39.2	+42 8	+46 4	25

* Sidereal time from elongation

TABLE XII.—VALUES OF $\frac{1}{2}(r' - r'')$

z_0 / $\frac{1}{2}(z'-z'')$	0°	10°	20°	25°	30°
′					
0	0.00	0 00	0 00	0 00	0 00
1	0 02	0 02	0 02	0 02	0.02
2	0 04	0 04	0 04	0 04	0 05
3	0 05	0 05	0 06	0 06	0 07
4	0 07	0 07	0.08	0 09	0 09
5	0 09	0 09	0 10	0 11	0.12
6	0 11	0 11	0 12	0.13	0.14
7	0 12	0 13	0 14	0.15	0 16
8	0 14	0 15	0 16	0 17	0.19
9	0 16	0 16	0.18	0 19	0.21
10	0 18	0.18	0 20	0.21	0.24
11	0 19	0 20	0 22	0.24	0 26
12	0 21	0 22	0 24	0.26	0 28
13	0 23	0 24	0 26	0 28	0 31
14	0 25	0 25	0.28	0 30	0 33
15	0 26	0 27	0.30	0.32	0.35
16	0 28	0 29	0.32	0.34	0 38
17	0 30	0 31	0 34	0 36	0 40
18	0 32	0 33	0 36	0 39	0 42
19	0 33	0 35	0 38	0 41	0 45
20	0 35	0 36	0 40	0 43	0.47

Greek Alphabet

A, α	alpha	N, ν	nu
B, β	beta	Ξ, ξ	xi
Γ, γ	gamma	O, o	omicron
Δ, δ	delta	Π, π	pi
E, ϵ	epsilon	P, ρ	rho
Z, ζ	zeta	Σ, σ, ς	sigma
H, η	eta	T, τ	tau
Θ, θ	theta	Υ, υ	upsilon
I, ι	iota	Φ, ϕ	phi
K, κ	kappa	X, χ	chi
Λ, λ	lambda	Ψ, ψ	psi
M, μ	mu	Ω, ω	omega

Abbreviations

R.A. or α = right ascension.
δ = declination.
t = hour angle.
h = altitude.
z = zenith distance.
z_m = meridian zenith distance.
A = azimuth.
ϕ = astronomical latitude.
λ = longitude.
T_a = apparent time.
T = mean or civil time.
E = equation of time.
θ = sidereal time.
G.C.T. = Greenwich Civil Time.
E.S.T. = Eastern Standard Time.
C.S.T. = Central Standard Time.
L.C.T. = Local Civil Time.
r = refraction.
p = parallax.
S = semidiameter of sun.

FORMS

Form for Observations

OBJECT: TIME FROM SINGLE ALTITUDE OF A STAR WITH THE ENGINEER'S TRANSIT

Date: ________________ Star: ________________

Transit No. ________________ Position: E. or W. of Meridian

Observer: ________________ Recorder: ________________

	Before Observing h m s	*After Observing* h m s
Clock Reading (L.C.T.)		
Watch Reading		
Correction, Watch to Clock		

OBSERVATIONS

Telescope	*Watch Time* h m s	*Vertical Circle* ° ′ ″
1. Direct		
2. Direct		
3. Direct		
4. Reversed		
5. Reversed		
6. Reversed		
Average Watch Time		Average Altitude
Correction, Watch to Clock		Index of Vertical Circle
Average Clock Time (T′)		h′

Form for Observations

OBJECT: TIME FROM SINGLE ALTITUDE OF A STAR WITH THE SEXTANT

Date: ____________ Star: ____________
Sextant No. ____________ Position: (E. or W. of Meridian) ____
Observer: ____________ Recorder: ____________

	Before Observing	*After Observing*
	h m s	h m s
Clock reading (L.C.T.)	______	______
Watch Reading	______	______
Correction, Watch to Clock	______	______

OBSERVATIONS

Index: 1. ____________ 3. ____________
2. ____________ 4. ____________
Average R ____________
i = ____________

	Double Altitude	*Watch Reading*
	° ′ ″	h m s
Roof A	______	______
	______	______
Roof B	______	______
	______	______

Average Double Altitudes ____________ Average Watch Reading ____________

Watch Correction ____________

i ____________

$2h'$ ____________ Average Clock Reading (T') ____________

h' ____________

FORM FOR COMPUTATION: TIME FROM SINGLE ALTITUDE OF A STAR

Date:________________ Computer:________________

$$\tan \tfrac{1}{2}t = \pm\sqrt{\frac{\sin \frac{1}{2}[z + (\phi - \delta)] \sin \frac{1}{2}[z - (\phi - \delta)]}{\cos \frac{1}{2}[z + (\phi + \delta)] \cos \frac{1}{2}[z - (\phi + \delta)]}}$$

	° ′ ″		
h'		$\log \sin \frac{1}{2}[z + (\phi - \delta)]$	
z'		$\log \sin \frac{1}{2}[z - (\phi - \delta)]$	
r		$\log \sec \frac{1}{2}[z + (\phi + \delta)]$	
ϕ		$\log \sec \frac{1}{2}[z - (\phi + \delta)]$	
δ			
		$\log \tan^2 \frac{1}{2}t$	
$\phi - \delta$		$\log \tan \frac{1}{2}t$	
			° ′ ″
z		$\frac{1}{2}t$	
$\phi + \delta$		$(\text{arc})t$	
			h m s
$z + (\phi - \delta)$		$(\text{time})t$	
$z - (\phi - \delta)$		α of star	
$z + (\phi + \delta)$		$\theta = \alpha + t$	
$z - (\phi + \delta)$		$T(\theta \rightarrow T)$	
$\frac{1}{2}[z + (\phi - \delta)]$		T'	
$\frac{1}{2}[z - (\phi - \delta)]$		ΔT	
$\frac{1}{2}[z + (\phi + \delta)]$			
$\frac{1}{2}[z - (\phi + \delta)]$			

Reduction of Sidereal into Civil Time

Sidereal Time of Greenwich 0^h C.T.__________

Reduction to Meridian__________

Local Sidereal Time for 0^h L.C.T.__________

Sidereal Time of Obs. (θ)__________

Sidereal Time after 0^h__________

Reduction to Mean Time Interval__________

Local Civil Time (T)__________

Form for Observations

OBJECT: TIME FROM SINGLE ALTITUDE OF THE SUN WITH THE ENGINEER'S TRANSIT

Date:__________________ Observer:__________________

Limb Observed:__________________ Recorder:__________________

Transit No. __________________

	Before Observing	*After Observing*
	h m s	h m s
Clock Reading (L.C.T.)		
Watch Reading		
Correction, Watch to Clock		

OBSERVATIONS

Telescope	*Watch Reading*	*Vertical Circle*
	h m s	° ′ ″
1. Direct		
2. Direct		
3. Direct		
4. Reversed		
5. Reversed		
6. Reversed		

Average Watch Reading______________ Average Altitude______________

Correction, Watch to Clock______________ Index Correction of Vertical Circle______________

Average Clock Reading (T')______________ h'______________

FORM FOR OBSERVATIONS

OBJECT: TIME FROM SINGLE ALTITUDE OF THE SUN WITH THE SEXTANT

Date:__________ Observer:__________

Limb Observed:__________ Recorder:__________

Sextant No.__________

	Before Observing h m s	*After Observing* h m s
Clock Reading (L.C.T.)		
Watch Reading		
Correction, Watch to Clock		

OBSERVATIONS

	On the Arc (R_1)		*Off the Arc* (R_2)
Index:			
1.		1.	
2.		2.	
Average R_1		Average R_2	
		i =	

	Double Altitude ° ′ ″		*Watch Reading* h m s
Roof A			
Roof B			
Average Double Altitude		Average Watch Reading	
i		Watch Correction	
$2h'$		Average	
h'		Clock Reading (T'')	

Form for Computation: Time from Single Altitude of the Sun

$$\tan \tfrac{1}{2}t = \pm\sqrt{\frac{\sin \frac{1}{2}[z + (\phi - \delta)] \sin \frac{1}{2}[z - (\phi - \delta)]}{\cos \frac{1}{2}[z + (\phi + \delta)] \cos \frac{1}{2}[z - (\phi + \delta)]}}$$

h'	° ′ ″	log sin $\frac{1}{2}[z + (\phi - \delta)]$	
z'		log sin $\frac{1}{2}[z - (\phi - \delta)]$	
r		log sec $\frac{1}{2}[z + (\phi + \delta)]$	
p		log sec $\frac{1}{2}[z - (\phi + \delta)]$	
S			
ϕ		log tan² $\frac{1}{2}t$	
δ		log tan $\frac{1}{2}t$	
			° ′ ″
		$\frac{1}{2}t$	
$\phi - \delta$		(arc) t	
z			h m s
$\phi + \delta$		(time) t	
		$T_a = 12^h + t$	
$z + (\phi - \delta)$		Equation of time (E)	
$z - (\phi - \delta)$		Local civil time (T)	
$z + (\phi + \delta)$		T'	
$z - (\phi + \delta)$		ΔT	
$\frac{1}{2}[z + (\phi - \delta)]$			
$\frac{1}{2}[z - (\phi - \delta)]$			
$\frac{1}{2}[z + (\phi + \delta)]$			
$\frac{1}{2}[z - (\phi + \delta)]$			

Form for Observations

OBJECT: DETERMINATION OF LATITUDE BY SINGLE ALTITUDE OF A STAR WITH ENGINEER'S TRANSIT, TIME BEING GIVEN

Transit No.____________ Observer:____________

Star Used:____________ Recorder:____________

Date:____________

SCHEDULE OF OBSERVATIONS

	Before Observations h m s	*After Observations* h m s
Clock		
Watch		
Correction, Watch to Clock		

Telescope	*Watch Reading* h m s	*Vertical Circle* ° ′ ″
Direct		
Direct		
Direct		
Reversed		
Reversed		
Reversed		

Average Watch Reading ______ Average Altitude h'______

Correction, Watch to Clock ______

Known Clock Correction______

Local Civil Time (T)______

Form for Computation

LATITUDE BY SINGLE ALTITUDE

Reduction of Observations

$$\tan F = \cot \delta \cos t; \sin (\phi + F) = \cos F \cos z \csc \delta$$

	h m s	Logarithms
Sidereal Time (θ)		$\cot \delta$
α		$\cos t$
t (time)		$\tan F$
t (arc)	° ′ ″	$\cos F$
h'		$\cos z$
z'		$\csc \delta$
r		$\sin (\phi + F)$
z		
δ		

	° ′ ″
$\phi + F$	
F	
ϕ	° ′ ″

Form of Observations

OBJECT: AZIMUTH FROM A CIRCUMPOLAR STAR WITH ENGINEER'S TRANSIT, TIME BEING GIVEN

Date: ____________ Recorder: ____________

Station: ____________ Observer: ____________

Transit No. ____________ Star Used: ____________

Position: E. or W. of Meridian

Before Observations	h m s	*After Observations*	h m s
Clock	____	Clock	____
Watch	____	Watch	____
Correction, Watch to Clock	____	Correction, Watch to Clock	____

Sight	*Watch Time*	*Horizontal Angle* *Vernier A*	*Vernier B*
		Telescope Direct	
	h m s	° ′ ″	° ′ ″
Mark	____	____	____
Star	____	____	____
Star	____	____	____
Mark	____	____	____
		Telescope Reversed	
Mark	____	____	____
Star	____	____	____
Star	____	____	____
Mark	____	____	____

Average Watch Time ____________ Average Reading on Mark (M) ____

Correction, Watch to Clock ____________

Known Clock Correction ____________ Average Reading on Star (K) ____

Local Civil Time of Observation ____________

Form of Computations

AZIMUTH FROM A CIRCUMPOLAR STAR, TIME BEING GIVEN

Date:________________ Computer:______________ Station:__________

$$\tan A = -\sin t \cot \delta \sec \phi \cdot \frac{1}{1 - a}$$

$$a = \cot \delta \tan \phi \cos t$$

	h m s or ° ′ ″	Logarithms
Local Civil Time of Observation		
Corresponding Sidereal Time		
α of Star		
t of Star (Time)		
t of Star (Arc)		
δ of Star		
ϕ, Latitude of Station		
log cot δ		
log tan ϕ		
log cos t		
log a		
log cot δ		
log sec ϕ		
log sin t		
log $[1 \div (1 - a)]$		
log tan A		n
A, Azimuth of Star from South Point		
Circle Reading on Star (K)		
Circle Reading on Mark (M)		
Difference ($K - M$)		
Azimuth of Mark (A_m)		

Form of Observations

OBJECT: AZIMUTH FROM THE SUN WITH ENGINEER'S TRANSIT, TIME BEING GIVEN

Date:__________ Recorder:__________

Station:__________ Observer:__________

Transit No.__________ Sun's Limb Observed:__________

Before Observations	h m s	*After Observations*	h m s
Clock		Clock	
Watch		Watch	
Correction, Watch to Clock		Correction, Watch to Clock	

Sight	*Watch Time*	*Horizontal Angle*	
		Vernier A	*Vernier B*
	h m s	Telescope Direct	
		° ′ ″	° ′ ″
Mark			
Sun			
Sun			
Mark			
		Telescope Reversed	
Mark			
Sun			
Sun			
Mark			

Average Watch Time__________ Average Reading on Mark (M)__________

Correction, Watch to Clock__________ Average Reading on Sun (K)__________

Known Clock Correction__________

Local Civil Time of Observation__________

	Direct	Reversed
	° ′	° ′
Approximate Altitude of Sun at Time of Direct and Reversed Readings		

FORM OF COMPUTATIONS: AZIMUTH FROM THE SUN, TIME BEING GIVEN

Date:________________ Computer: ____________ Station: ______________

$$\tan A = -\sin t \cot \delta \sec \phi \cdot \frac{1}{1-a}; \; a = \cot \delta \tan \phi \cos t.$$

	h m s or ° ′ ″	Logarithms
Local Civil Time of Observation		
Equation of Time........		
Apparent Time of Observation		
t = Hour Angle of Sun (Time). ..		
t = Hour Angle of Sun (Arc).		
ϕ = Latitude of Station.... .		
δ of Sun (for time of observation) .		
cot δ		
tan ϕ		
cos t.		
a ..		
cot δ.		
sec ϕ.		
sin t..		
$1 \div (1 - a)$		
$-$ tan A.		
A, Azimuth of Sun from South Point		
S = semidiameter of Sun from Ephemeris.. .		
sin z (z observed or computed zenith distance)		
s = correction for horizontal angle		
Horizontal angle reading on sun.		
Corrected horizontal angle (s correction) .		
Difference: Mark $-$ sun . .		
Azimuth of mark ...		

Form of Observations

OBJECT: AZIMUTH FROM THE SUN WITH ENGINEER'S TRANSIT, TIME NOT BEING GIVEN

Date:________________ Recorder:________________

Station:________________ Observer:________________

Transit No.________________ Limbs Observed:________________

Sight	Approximate Time	Vertical Angle	Horizontal Angle	
			A	*B*
		Telescope Direct		
	h m s	° ′ ″	° ′ ″	° ′ ″
Mark				
Sun				
Sun				
Mark				
		Telescope Reversed		
		° ′ ″	° ′ ″	° ′ ″
Mark				
Sun				
Sun				
Mark				

Average Time________________

Approximate G.C.T.
of Observation________________

Average, Vertical Angle________________

Average, Reading on Mark (*M*)________________

Average, Reading on Sun________________

Form of Computations: Azimuth from the Sun, Time Not Being Given

$$\cot \tfrac{1}{2}A = \pm\sqrt{\frac{\cos \tfrac{1}{2}(z + [\phi + \delta]) \sin \tfrac{1}{2}(z + [\phi - \delta])}{\cos \tfrac{1}{2}(z - [\phi + \delta]) \sin \tfrac{1}{2}(z - [\phi - \delta])}}$$

	° ′ ″		° ′ ″	Logarithms
h'		$\cos \tfrac{1}{2}(z + [\phi + \delta])$		
z'		$\sin \tfrac{1}{2}(z + [\phi - \delta])$		
r		$\sec \tfrac{1}{2}(z - [\phi + \delta])$		
p		$\csc \tfrac{1}{2}(z - [\phi - \delta])$		
S		$\cot^2 \tfrac{1}{2}A$		
ϕ		$\cot \tfrac{1}{2}A$		
δ		$\tfrac{1}{2}A$		
$\phi - \delta$		A		
z				
$\phi + \delta$		$s = \dfrac{S}{\sin z}$		
$z + [\phi + \delta]$ $z + [\phi - \delta]$		Observed horizontal reading on sun		
$z - [\phi + \delta]$ $z - [\phi - \delta]$		Corrected horizontal reading on sun (K)		
$\tfrac{1}{2}(z + [\phi + \delta])$ $\tfrac{1}{2}(z + [\phi - \delta])$ $\tfrac{1}{2}(z - [\phi + \delta])$ $\tfrac{1}{2}(z - [\phi - \delta])$		Observed horizontal reading on mark (M)		
		Required azimuth, $A_m = A - (K - M)$		

STAR MAPS

CHART I

STARS ABOUT THE NORTH POLE

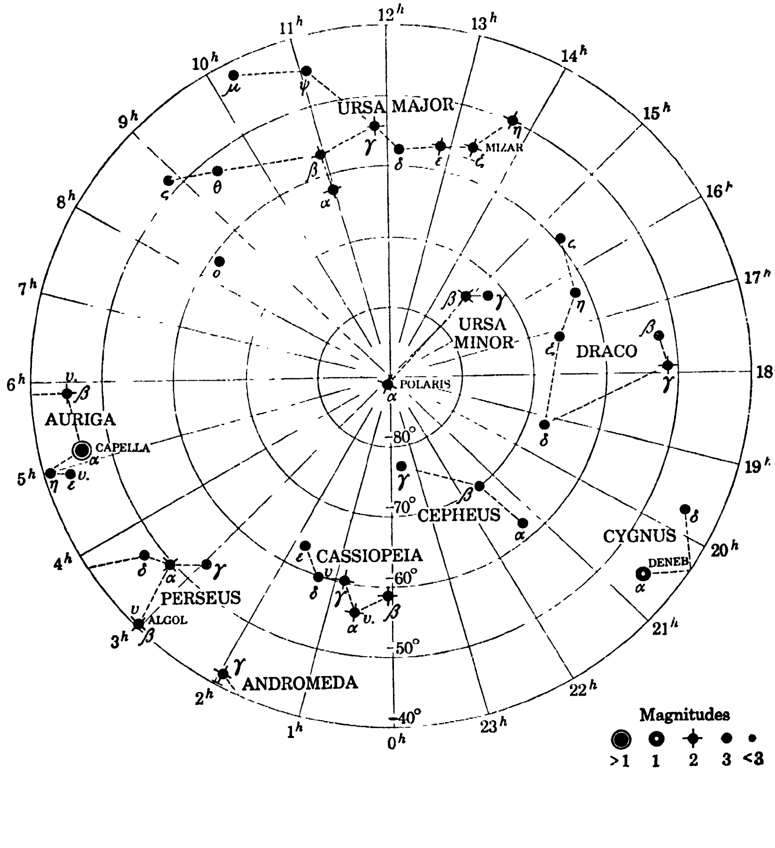


CHART II

STARS BETWEEN DECLINATION 50° NORTH AND 50° SOUTH

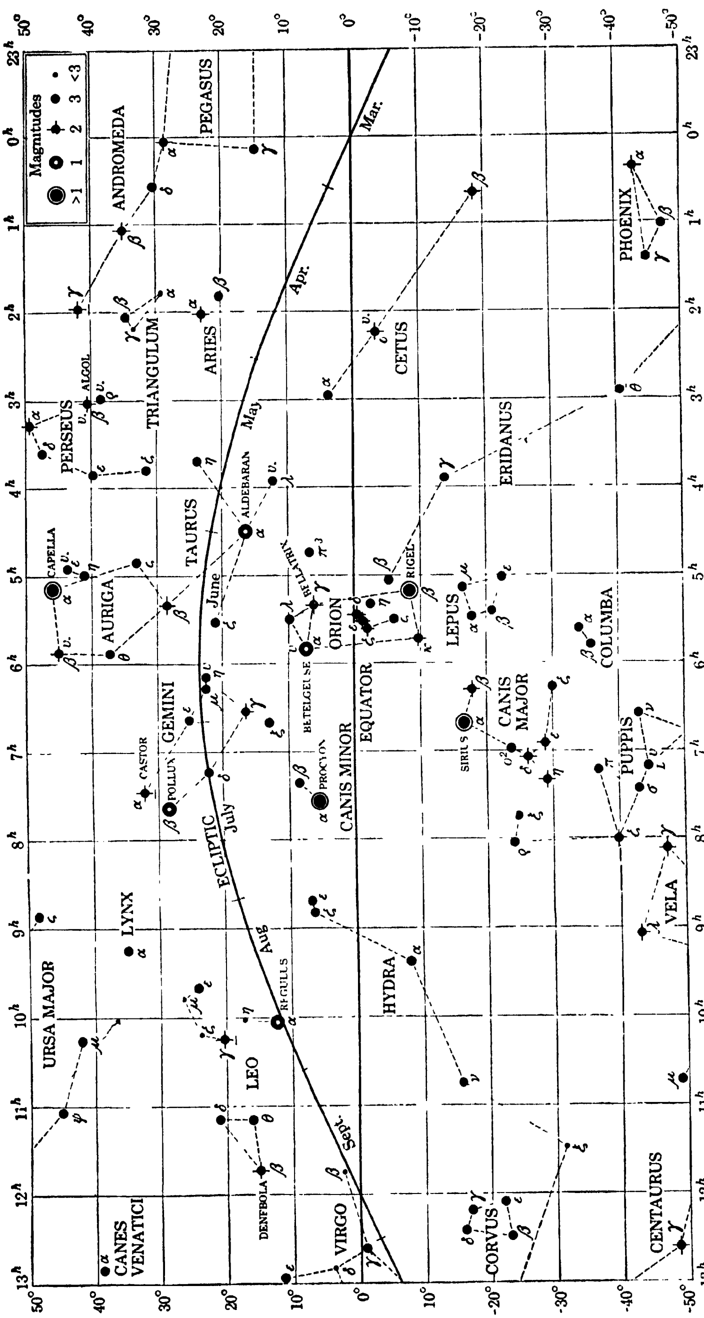


CHART III
STARS BETWEEN DECLINATION 50° NORTH AND 50° SOUTH

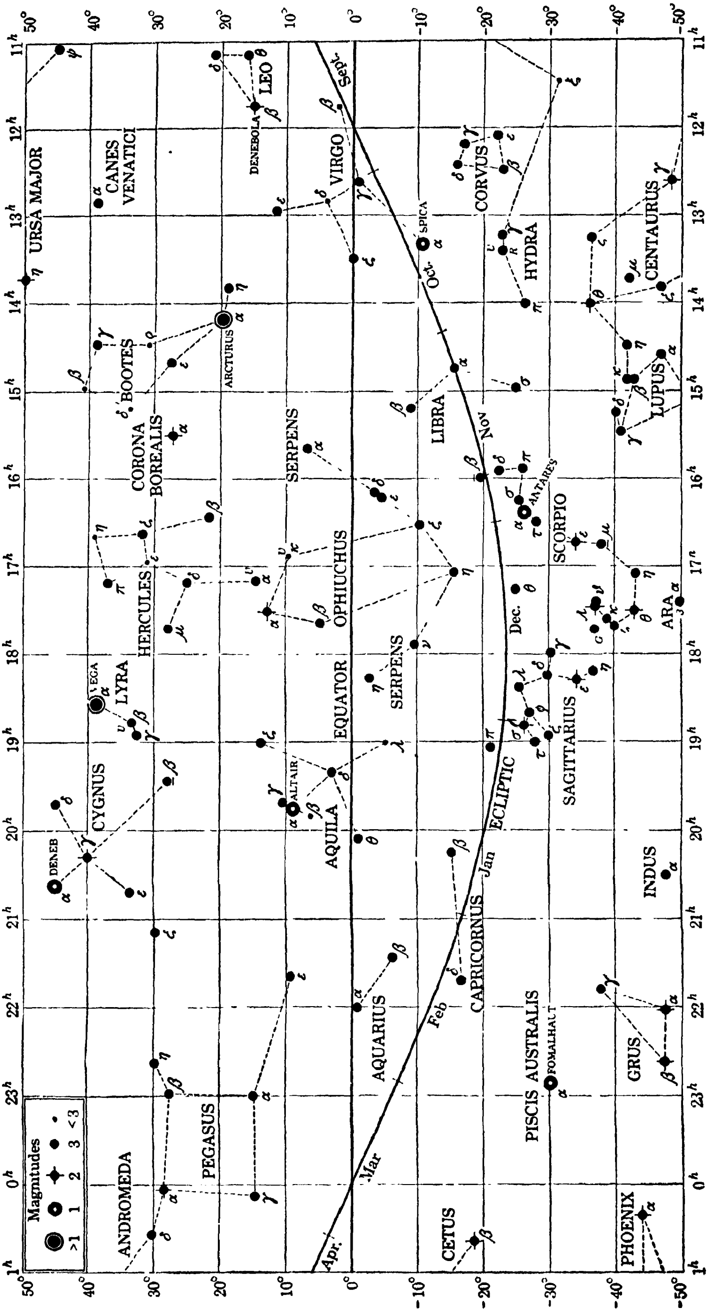
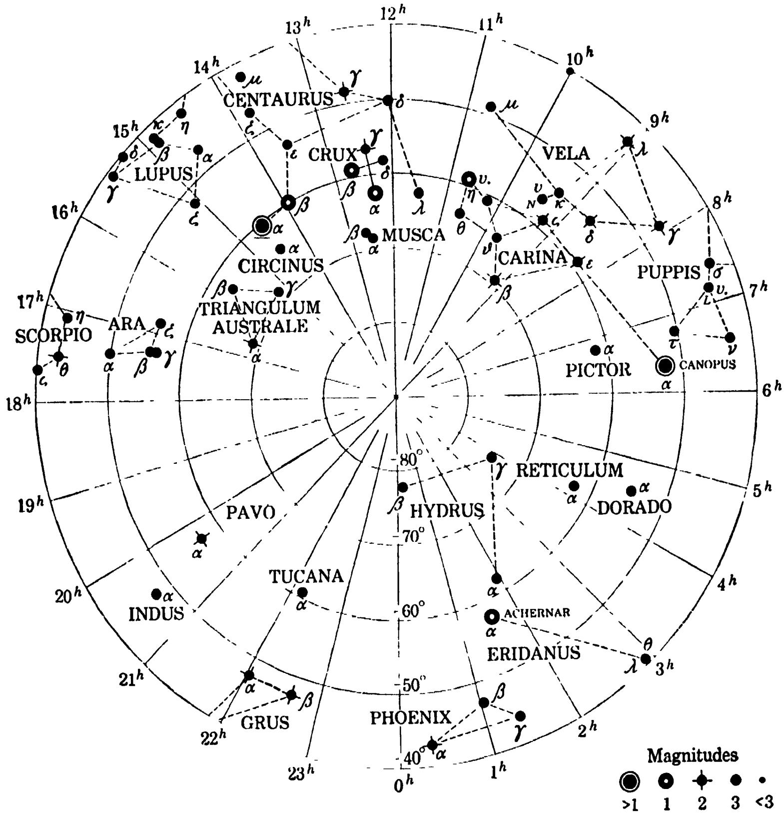


CHART IV
STARS ABOUT THE SOUTH POLE



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